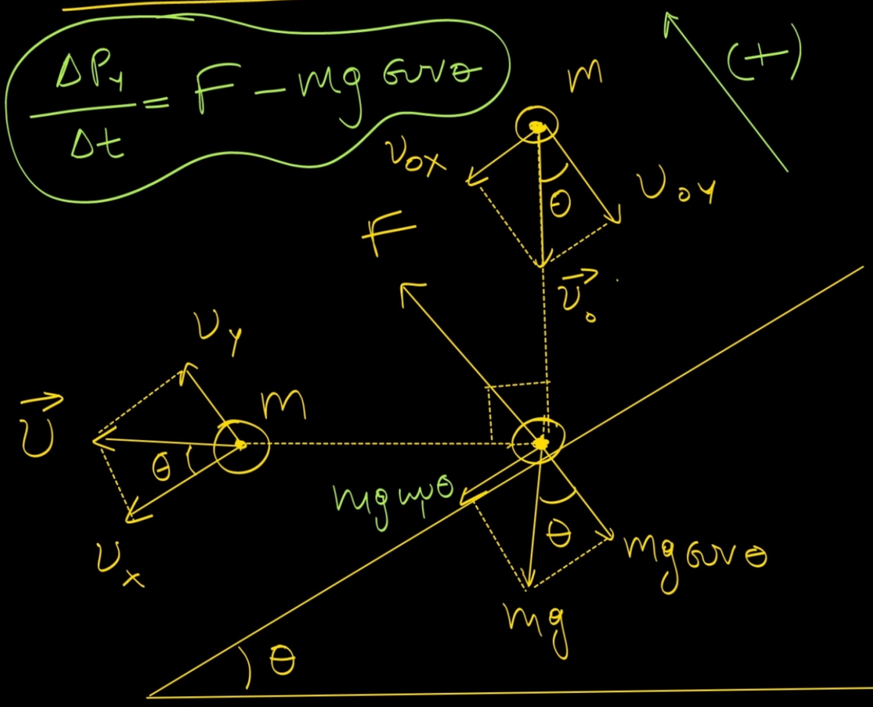
$$\Sigma \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

Αθ. 3.58) (ΣκΒΒ)

$$\frac{\Delta p_y}{\Delta t} = F - mg\,\sigma\upsilon\nu\theta$$



α) Η δύναμη $\vec{F}$ είναι κάθετη στο κεκλιμένο, δηλαδή κατά μήκος του άξονα Υ, οπότε η Χ-συνιστώσα της ορμής δεν αλλάζει.

$$P_{x,\pi\rho\iota\nu} = P_{x,\mu\varepsilon\tau\acute{\alpha}} \Rightarrow$$

$$m \cdot \upsilon_{0x} = m \cdot \upsilon_x \Rightarrow$$

$$\upsilon_0 \cdot \eta\mu\theta = \upsilon \cdot \sigma\upsilon\nu\theta \Rightarrow$$

$$40 \cdot \frac{3}{5} = \upsilon \cdot \frac{4}{5} \Rightarrow$$

$$\upsilon = 30\,m/s.$$

β) Ποσοστό απώλειας ΚΕ =

$$\frac{\Delta K}{K_{\pi\rho\iota\nu}} \cdot 100 =$$

$$= \frac{K_{\mu\varepsilon\tau\acute{\alpha}} - K_{\pi\rho\iota\nu}}{K_{\pi\rho\iota\nu}} \cdot 100 =$$

$$= \frac{\frac{1}{2}m\upsilon^2 - \frac{1}{2}m\upsilon_0^2}{\frac{1}{2}m\upsilon_0^2} \cdot 100 =$$

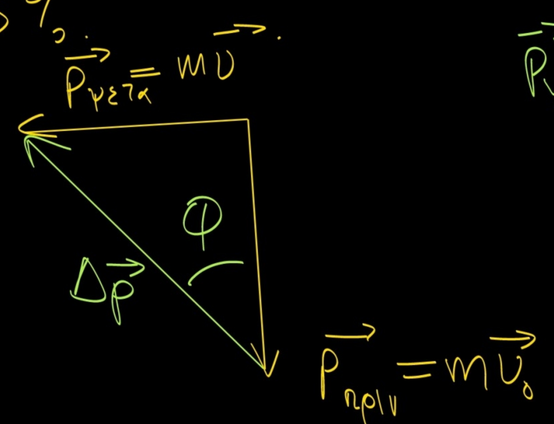
$$= \frac{\upsilon^2 - \upsilon_0^2}{\upsilon_0^2} \cdot 100 = \frac{900 - 1600}{1600} \cdot 100 = \frac{-700}{16} = -\frac{350}{8} = -\frac{175}{4}$$

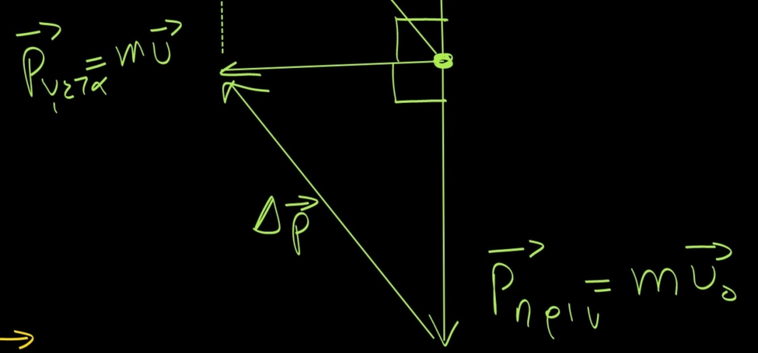
$$\frac{\Delta K}{K_{\pi\rho\iota\nu}} \cdot 100 = \text{ποσοστιαία μεταβολή } KE$$

$$-\frac{17,5}{4} = -\frac{87,5}{2} = -43,75.$$

Άρα η ΚΕ μειώθηκε κατά 43,75%.

β)





$$\Delta p^2 = (mv)^2 + (mv_0)^2 = 1600 + 900 \Rightarrow$$

$$\Delta p = 50\,kg \cdot m/s.$$

(μέτρο)

$$\Delta p = |\Delta\vec{p}|$$

$$\varepsilon\varphi\varphi = \frac{mv}{mv_0} = \frac{3}{4}$$

(κατεύθυνση)

δ) $\Sigma F_y = \frac{\Delta P_y}{\Delta t} \Rightarrow$

$F - mg\,\text{συν}\theta = \frac{P_{y,\text{μετά}} - P_{y(\text{πριν})}}{\Delta t} \Rightarrow$

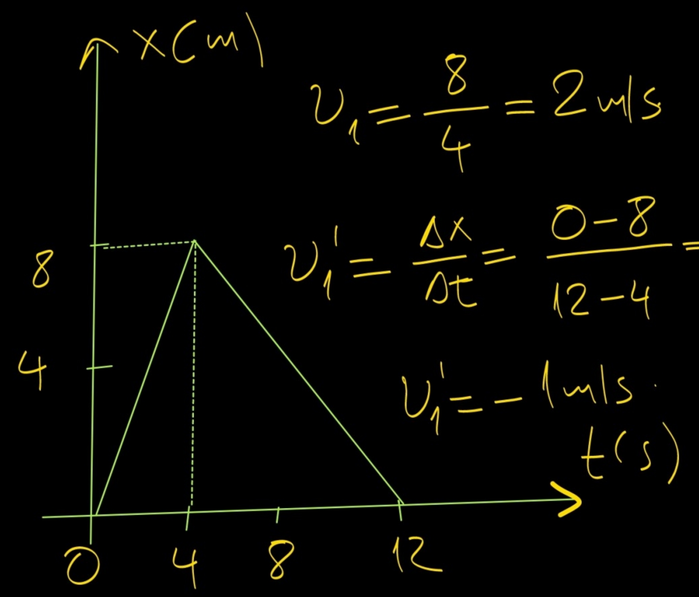
$F - mg \cdot \text{συν}\theta = \frac{m \cdot v_y - (-m \cdot v_{0y})}{\Delta t} \Rightarrow$

$F - mg\,\text{συν}\theta = \frac{m(v_y + v_{0y})}{\Delta t} \Rightarrow$

$F - mg\,\text{συν}\theta = \frac{m(v \cdot \text{ημ}\theta + v_0 \cdot \text{συν}\theta)}{\Delta t} \Rightarrow 508 - 1 \cdot 10 \cdot \frac{4}{5} = \frac{1 \cdot \left(30 \cdot \frac{3}{5} + 40 \cdot \frac{4}{5}\right)}{\Delta t} \Rightarrow$

$508 - 8 = \frac{\frac{250}{5}}{\Delta t} \Rightarrow 500 = \frac{50}{\Delta t} \Rightarrow \Delta t = 0,1\,\text{sec}.$

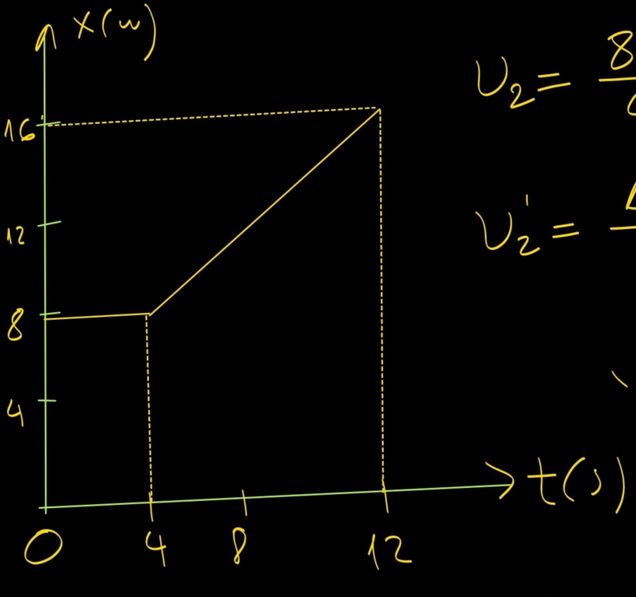
Ασκ. 2.61.



$$v_1 = \frac{8}{4} = 2\ m/s$$

$$v_1' = \frac{\Delta x}{\Delta t} = \frac{0-8}{12-4} \Rightarrow$$

$$v_1' = -1\ m/s$$



$$v_2 = \frac{8-8}{4-0} = 0\ m/s$$

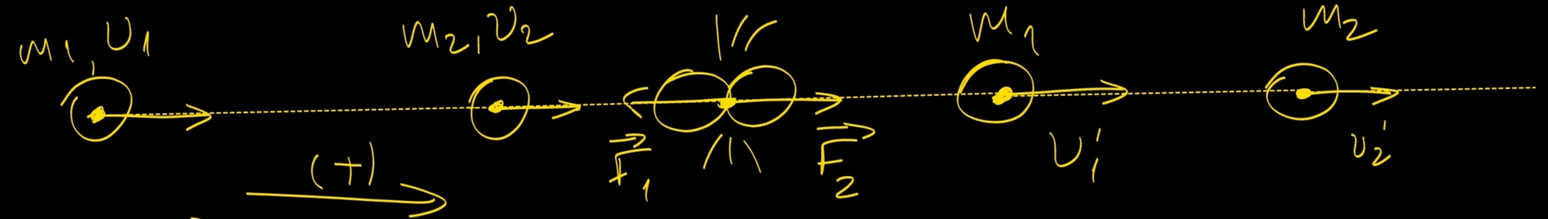
$$v_2' = \frac{\Delta x}{\Delta t} = \frac{16-8}{12-4} = 1\ m/s$$

Άρα: $v_1 + v_1' = 2 - 1 = 1\ m/s$

$v_2 + v_2' = 0 + 1 = 1\ m/s$

Συνεπώς: $v_1 + v_1' = v_2 + v_2'$, οπότε η ταχύτητα είναι ελαστική

$$v_1' - v_2' = -(v_1 - v_2)$$



$(u_1 > u_2)$

$P_{\pi\rho\iota\nu} = P_{\mu\varepsilon\tau\acute{\alpha}} \Rightarrow m_1u_1 + m_2u_2 = m_1u_1' + m_2u_2'$ (1)

$K_{\pi\rho\iota\nu} = K_{\mu\varepsilon\tau\acute{\alpha}} \Rightarrow \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1u_1'^2 + \frac{1}{2}m_2u_2'^2$ (2)

$(1) \Rightarrow m_1u_1 - m_1u_1' = m_2u_2' - m_2u_2 \Rightarrow m_1(u_1 - u_1') = m_2(u_2' - u_2)$ (3)

$(2) \Rightarrow m_1u_1^2 - m_1u_1'^2 = m_2u_2'^2 - m_2u_2^2 \Rightarrow m_1(u_1^2 - u_1'^2) = m_2(u_2'^2 - u_2^2) \Rightarrow$

$m_1(u_1 - u_1')(u_1 + u_1') = m_2(u_2' - u_2)(u_2' + u_2)$ (4)

Διαιρούμε την (4) με την (3):

$$\frac{(4)}{(3)} \Rightarrow \frac{\cancel{m_1}(v_1-v_1')(v_1+v_1')}{\cancel{m_1}(v_1-v_1')} = \frac{\cancel{m_2}(v_2'-v_2)(v_2'+v_2)}{\cancel{m_2}(v_2'-v_2)} \Rightarrow$$

$$v_1+v_1' = v_2'+v_2 \quad (5)$$

Η (5) γράφεται: $v_1'-v_2' = -v_1+v_2$ (6)

Και η ΑΔΟ: $m_1v_1'+m_2v_2' = m_1v_1+m_2v_2$

Για να βρούμε τα v_1' και v_2' λύνουμε το σύστημα:

$$\left.\begin{array}{l} v_1'-v_2' = -v_1+v_2 \\ m_1v_1'+m_2v_2' = m_1v_1+m_2v_2 \end{array}\right| \begin{array}{l} \cdot m_2 \\ \cdot 1 \end{array} \Rightarrow \left|\begin{array}{l} m_2v_1'-\cancel{m_2v_2'} = -m_2v_1+m_2v_2 \\ m_1v_1'+\cancel{m_2v_2'} = m_1v_1+m_2v_2 \end{array}\right| (+) \Rightarrow$$

$$(m_1+m_2)v_1' = (m_1-m_2)v_1+2m_2v_2 \Rightarrow$$

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

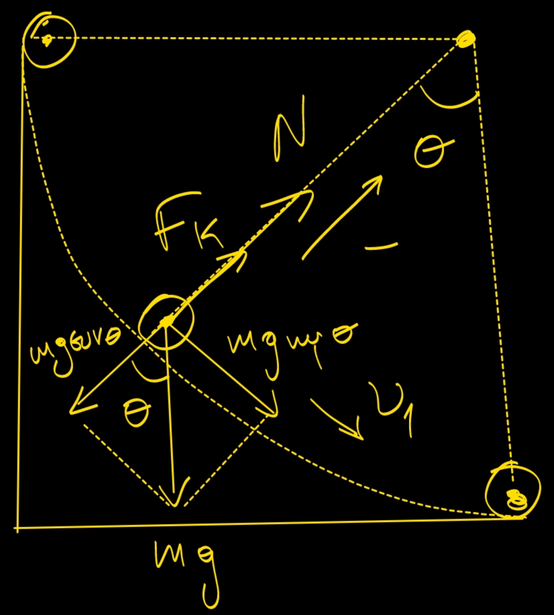
$$v_1' - v_2' = -v_1 + v_2 \Rightarrow$$

$$v_2' = v_1' + v_1 - v_2 = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2 + \frac{m_1 + m_2}{m_1 + m_2} v_1 - \frac{m_1 + m_2}{m_1 + m_2} v_2 =$$

$$= \frac{2m_1}{m_1 + m_2} v_1 + \frac{2m_2 - m_1 - m_2}{m_1 + m_2} v_2 \Rightarrow$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

Άσκηση



$m = 20\,g$ $\quad U_1 = 4\,m/s$.

$R = 1{,}6\,m$

$\theta = 60^\circ$

α) Το μέτρο της επιτάχυνσης.

$$m \cdot \alpha_{εφαπτ} = mg\eta\mu\theta \Rightarrow$$

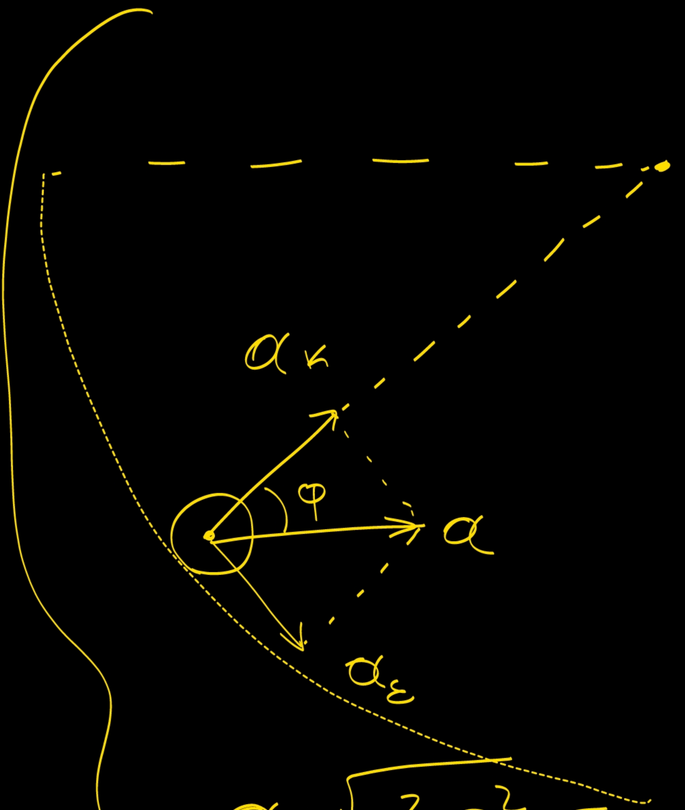
$$\alpha_\varepsilon = g \cdot \eta\mu\theta$$

$$\Sigma F_{κεντ} = N - mg\sigma\upsilon\nu\theta \Rightarrow$$

$$m\alpha_K = N - mg\sigma\upsilon\nu\theta$$

$$\alpha_K = \frac{U_1^2}{\ell} = \frac{16}{1{,}6} = 10\,m/s^2$$

$$\alpha_\varepsilon = 10 \cdot \eta\mu 60^\circ = 10\frac{\sqrt{3}}{2} = 5\sqrt{3}\,m/s^2$$

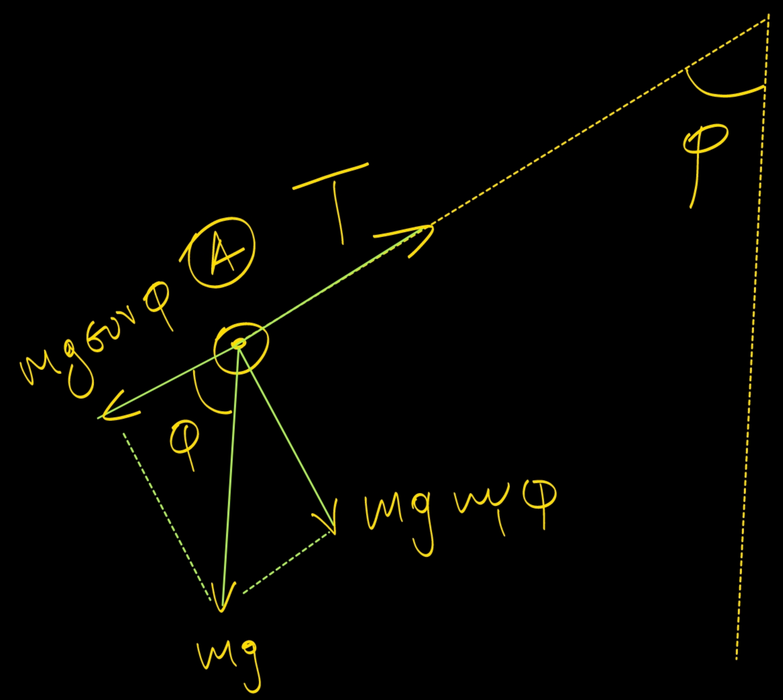


$$\alpha = \sqrt{\alpha_\varepsilon^2 + \alpha_K^2} =$$

$$= \sqrt{25 \cdot 3 + 100} =$$

$$= \sqrt{175} = 5\sqrt{7}\,m/s^2$$

β) $N = m\alpha_K + mg\sigma\upsilon\nu\theta = 0{,}02 \cdot 10 + 0{,}02 \cdot 10 \cdot \frac{1}{2} = 0{,}3\,N$



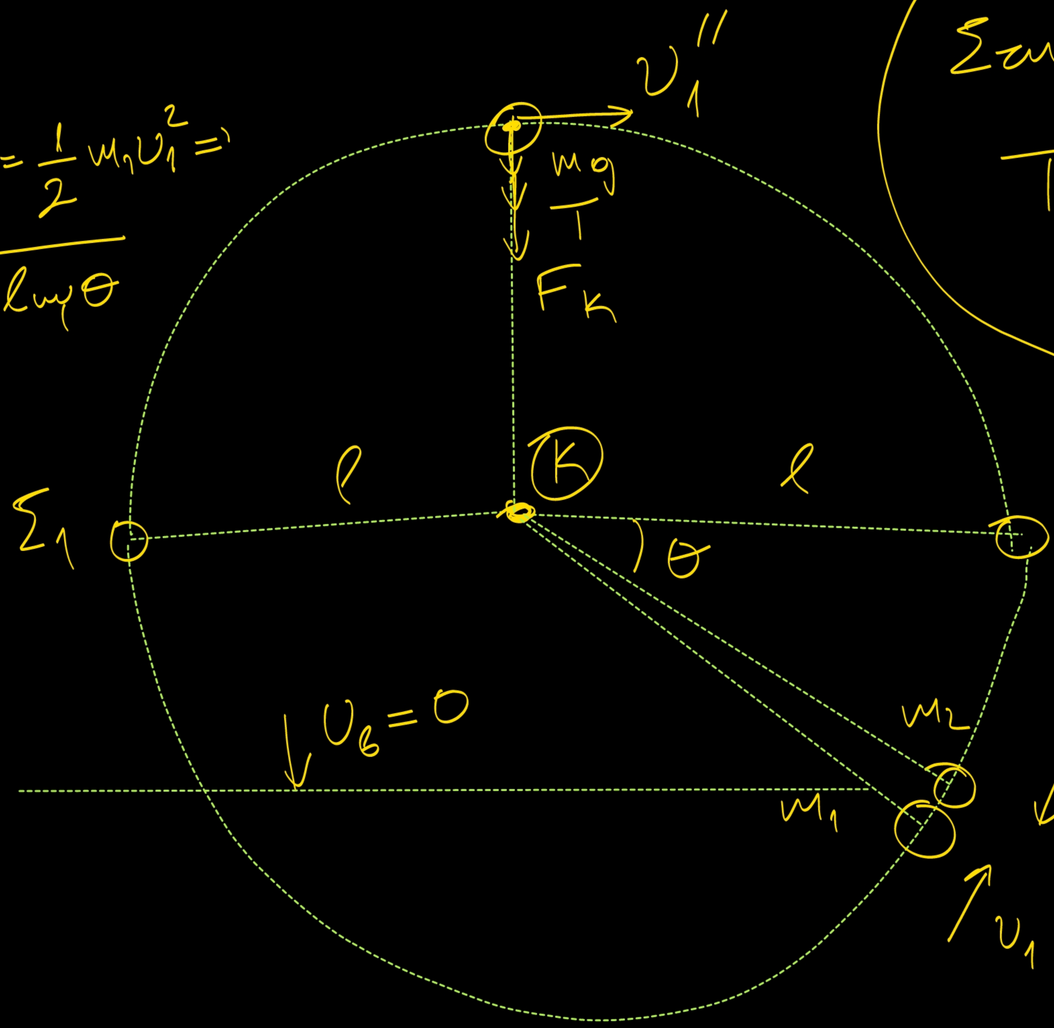
Στη θέση (A):

$$T = mg\sigma\upsilon\nu\varphi$$

και $$\frac{\Delta p}{\Delta t} = \Sigma F = mg\eta\mu\varphi \Rightarrow$$

$$10\sqrt{3} = 20 \cdot \eta\mu\varphi \Rightarrow$$

$$\eta\mu\varphi = \frac{\sqrt{3}}{2} \Rightarrow \varphi = 60^\circ.$$



$m_1 g \ell \sigma\upsilon\nu\theta = \frac{1}{2} m_1 U_1^2 \Rightarrow$

$U_1 = \sqrt{2g\ell\sigma\upsilon\nu\theta}$

Στην πάνω θέση έχουμε:

$T + mg = F_K \Rightarrow$

$T + mg = \frac{m U_1''^2}{\ell}$

Για να γίνει αναπύλωση πρέπει να είναι $T = 0$

οπότε $U_1'' = \sqrt{g\ell}$.

$U_1 = U_2 = \sqrt{2g\ell\sigma\upsilon\nu\theta}$

$T > 0$ γίνεται αναπύλωση

$\frac{m U_1''^2}{\ell} - mg > 0 \Rightarrow U_1'' > \sqrt{g\ell}$.