Ασκ. 2.182 (Μ-Θ)

$$\left(v_2' = \frac{2m_1}{m_1+m_2}v_1 = \frac{4}{2+m_2}\cdot v_1\right)$$

α) $K_{2,\text{μετά}} = \frac{8}{9}K_{1,\text{πριν}}$ (1ος τρόπος)

$\Updownarrow$

$K_2' = \frac{8}{9}K_1 \Rightarrow$

$\frac{1}{2}m_2 v_2'^2 = \frac{8}{9}\cdot\frac{1}{2}m_1 v_1^2 \Rightarrow$

$m_2 v_2'^2 = \frac{8}{9}m_1 v_1^2 \Rightarrow m_2\cdot\left(\frac{4}{2+m_2}\right)^2 \cancel{v_1^2} = \frac{8}{9}m_1\cancel{v_1^2} \Rightarrow$

$m_2\cdot\frac{16}{(2+m_2)^2} = \frac{16}{9} \Rightarrow 9m_2 = (2+m_2)^2 \Rightarrow$

$9m_2 = 4 + 4m_2 + m_2^2 \Rightarrow$

$m_2^2 - 5m_2 + 4 = 0$

$\Delta = 25 - 16 = 9$

$m_2 = \frac{5\pm 3}{2} \rightleftarrows \frac{8}{2} = 4\,\text{kg}$

$\frac{2}{2} = 1\,\text{kg}.$

Δε μεταβάλλεται η μάζα (δηλαδή η φορά) της m_1 δε μεσωπική ελαστική κρούση, άρα $m_1 > m_2 \Rightarrow$

$m_2 = 1\,\text{kg}.$

(2ος τρόπος)

$K_2' = \frac{9}{9} K_1$

Άρα: $K_1' = \frac{1}{9} K_1 \Rightarrow$

$\frac{1}{2} m_1 v_1'^2 = \frac{1}{9} \cdot \frac{1}{2} m_1 v_1^2 \Rightarrow$

$v_1'^2 = \frac{v_1^2}{9} \Rightarrow v_1' = \pm \frac{v_1}{3}$

Η Σ_1 δεν αλλάξει φορά

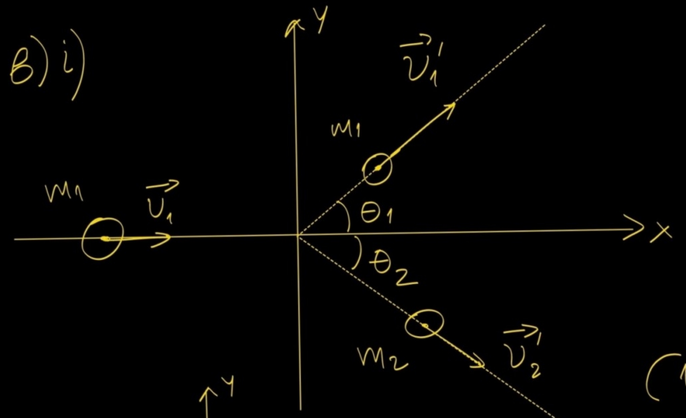
άρα: $v_1' = + \frac{v_1}{3} \Rightarrow \frac{m_1 - m_2}{m_1 + m_2} v_1 = \frac{1}{3} v_1 \Rightarrow \frac{m_1 - m_2}{m_1 + m_2} = \frac{1}{3} \Rightarrow$

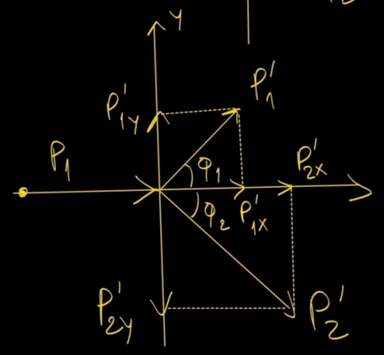
$\rightarrow 3m_1 - 3m_2 = m_1 + m_2 \Rightarrow$

$2m_1 = 4m_2 \Rightarrow$

$4 = 4m_2 \Rightarrow m_2 = 1\,Kg$

B) i)





$$P_{x,\text{πριν}} = P_{x,\text{μετά}} \Rightarrow$$

$$P_1 = P_1' \sigma\upsilon\nu\varphi_1 + P_2' \sigma\upsilon\nu\varphi_2 \quad (1)$$

$$P_{y,\text{πριν}} = P_{y,\text{μετά}} \Rightarrow$$

$$0 = P_{1y}' - P_{2y}' \Rightarrow 0 = P_1' \eta\mu\varphi_1 - P_2' \eta\mu\varphi_2 \quad (2)$$

$$(1): P_2' \sigma\upsilon\nu\varphi_2 = P_1 - P_1' \sigma\upsilon\nu\varphi_1 \Big| \Rightarrow$$

$$P_2' \cdot \eta\mu\varphi_2 = P_1' \eta\mu\varphi_1$$

$$P_2'^2 \sigma\upsilon\nu^2\varphi_2 + P_2'^2 \eta\mu^2\varphi_2 = (P_1 - P_1' \sigma\upsilon\nu\varphi_1)^2 + P_1'^2 \eta\mu^2\varphi_1 \Rightarrow$$

$$P_2'^2 (\sigma\upsilon\nu^2\varphi_2 + \eta\mu^2\varphi_2) = P_1^2 - 2P_1P_1'\sigma\upsilon\nu\varphi_1 + P_1'^2 \sigma\upsilon\nu^2\varphi_1 + P_1'^2 \eta\mu^2\varphi_1 \Rightarrow$$

$$p_2'^2 = p_1^2 - 2p_1p_1'\sigma\upsilon\nu\varphi_1 + p_1'^2\cdot(\sigma\upsilon\nu^2\varphi_1 + \eta\mu^2\varphi_1) \Rightarrow$$

$$p_2'^2 = p_1^2 - 2p_1p_1'\sigma\upsilon\nu\varphi_1 + p_1'^2$$

$$K = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$$

$$K = \frac{1}{2}mv\cdot v \Rightarrow \boxed{K = \frac{1}{2}p\cdot v}$$

$$\frac{p_1^2}{2m_1} = \frac{p_1'^2}{2m_1} + \frac{p_2'^2}{2m_2} \Rightarrow \frac{p_1^2 - p_1'^2}{m_1} = \frac{p_2'^2}{m_2} \Rightarrow p_2'^2 = \frac{m_2}{m_1}\left(p_1^2 - p_1'^2\right)$$

$$\frac{m_2}{m_1}\left(p_1^2 - p_1'^2\right) = p_1^2 - 2p_1p_1'\sigma\upsilon\nu\varphi_1 + p_1'^2 \Rightarrow$$

$$m_2p_1^2 - m_2p_1'^2 = m_1p_1^2 - 2m_1p_1p_1'\sigma\upsilon\nu\varphi_1 + m_1p_1'^2 \Rightarrow$$

$$(m_1+m_2)p_1'^2 - 2m_1p_1p_1'\sigma\upsilon\nu\varphi_1 + (m_1-m_2)p_1^2 = 0$$

Τριώνυμο ως προς p_1' με

$\alpha = m_1+m_2$

$\beta = -2m_1p_1\sigma\upsilon\nu\varphi_1$ και $\gamma = +(m_1-m_2)p_1^2$

Άρα το τριώνυμο είναι:

$$(m_1+m_2)p_1'^2 - 2m_1p_1p_1'\sigma\upsilon\nu\varphi_1 + (m_1-m_2)p_1^2 = 0$$

με $\alpha = m_1+m_2$

$$\beta = -2m_1p_1\sigma\upsilon\nu\varphi_1$$

$$\gamma = (m_1-m_2)p_1^2$$

Το τριώνυμο πρέπει να έχει λύση για κάθε p_1', άρα:

$$\Delta \geq 0 \Rightarrow \beta^2 - 4\alpha\gamma \geq 0 \Rightarrow$$

$$4m_1^2p_1^2\sigma\upsilon\nu^2\varphi_1 - 4(m_1+m_2)(m_1-m_2)p_1^2 \geq 0 \Rightarrow$$

$$m_1^2\sigma\upsilon\nu^2\varphi_1 - (m_1^2-m_2^2) \geq 0 \Rightarrow$$

$$\sigma\upsilon\nu^2\varphi_1 \geq \frac{m_1^2-m_2^2}{m_1^2} \Rightarrow \sigma\upsilon\nu^2\varphi_1 \geq \frac{4-1}{4} \Rightarrow$$

$$\sigma\upsilon\nu^2\varphi_1 \geq \frac{3}{4} \Rightarrow 1-\eta\mu^2\varphi_1 \geq \frac{3}{4} \Rightarrow$$

$$1-\frac{3}{4} \geq \eta\mu^2\varphi_1 \Rightarrow \eta\mu^2\varphi_1 \leq \frac{1}{4} \Rightarrow |\eta\mu\varphi_1| \leq \frac{1}{2} \Rightarrow$$

$$-\frac{1}{2} \leq \eta\mu\varphi_1 \leq \frac{1}{2} \Rightarrow -\eta\mu\frac{\pi}{6} \leq \eta\mu\varphi_1 \leq \eta\mu\frac{\pi}{6} \Rightarrow$$

$$-\frac{\pi}{6} \leq \varphi_1 \leq \frac{\pi}{6} \Rightarrow \varphi_{1,max} = \frac{\pi}{6} \text{ rad}$$

ii) Έστω $\varphi_1 = 30^\circ$.

$$p_1 = p_1' \sigma\upsilon\nu\varphi_1 + p_2' \sigma\upsilon\nu\varphi_2$$

$$p_1' \eta\mu\varphi_1 = p_2' \eta\mu\varphi_2$$

$$p_1 = m_1 v_1 = 20\ \text{kg}\cdot\text{m/s}.$$

$$\varphi_1 = 30^\circ$$

$$K_1 = \frac{p_1^2}{2m_1} = \frac{400}{2\cdot 2} = 100\ \text{J}$$

$$(m_1+m_2)p_1'^2 - 2m_1 p_1 p_1' \sigma\upsilon\nu\varphi_1 + (m_1-m_2)p_1^2 = 0 \Rightarrow$$

$$3p_1'^2 - 2\cdot 2\cdot 20\, p_1' \cdot \sigma\upsilon\nu 30^\circ + (2-1)\cdot 20^2 = 0 \Rightarrow$$

$$3p_1'^2 - 80p_1'\cdot\frac{\sqrt{3}}{2}p_1' + 400 = 0 \Rightarrow$$

$$(\sqrt{3}p_1')^2 - 40\sqrt{3}p_1' + 20^2 = 0 \Rightarrow$$

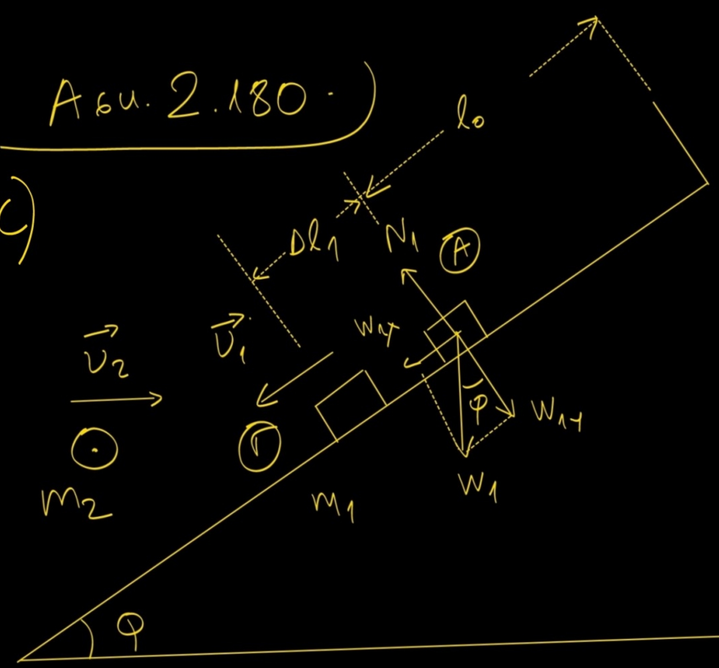
$$(\sqrt{3}p_1' - 20)^2 = 0 \Rightarrow p_1' = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3}\ \text{kg}\cdot\text{m/s}.$$

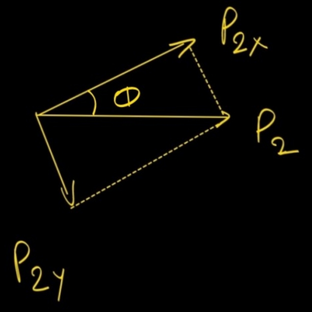
$$K_1' = \frac{p_1'^2}{2m_1} = \frac{\frac{400\cdot 3}{9}}{2\cdot 2} = \frac{400}{4\cdot 3} = 33{,}3\ \text{J}$$

$K_2' = 66{,}7$ Joule άρα: Ποσοστό $= \frac{K_2'}{K_1}\cdot 100 = \frac{66{,}7}{100}\cdot 100 = 66{,}7\%$

Ασκ. 2.180 ·)

α)





$P_{x,\text{πριν}} = P_{x,\text{μετά}} \Rightarrow$

$P_{2x} = (m_1+m_2)V \Rightarrow$

$m_2 v_2 \sigma\upsilon\nu\varphi = (m_1+m_2)V \Rightarrow$

$2 \cdot 4 \cdot \sigma\upsilon\nu 30^\circ = 4V \Rightarrow$

$2 \cdot \frac{\sqrt{3}}{2} = V \Rightarrow V = \sqrt{3}$ m/s.

με φορά προς τα πάνω.

ΘΜΚΕ: Α→Γ: $K_\Gamma - K_A = W_{F_{\Sigma\Lambda}(A\to\Gamma)} + W_{w_{1x}(A\to\Gamma)} \Rightarrow$

$\frac{1}{2}m_1 v_1^2 - 0 = 0 - \frac{1}{2}k\cdot\Delta l_1^2 + m_1 g\eta\mu\varphi\cdot\Delta l_1 \Rightarrow \frac{1}{2}\cdot 2\cdot v_1^2 = -\frac{1}{2}\cdot 100\cdot 0{,}2^2 + 2\cdot 10\cdot\frac{1}{2}\cdot 0{,}2 \Rightarrow$

$v_1^2 = -2+2 \Rightarrow v_1 = 0$ m/s.

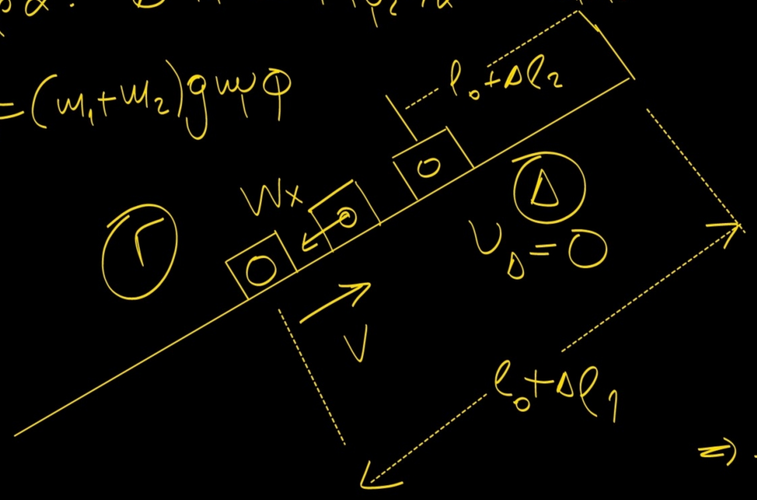
β) $K_{2,\pi\rho\iota\nu} = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \cdot 2 \cdot 4^2 = 16$ Joule.

$K_{1,\pi\rho\iota\nu} = 0$.

$K_{\upsilon\sigma\tau\acute{\alpha}} = \frac{1}{2}(m_1+m_2)V^2 = \frac{1}{2} \cdot 4 \cdot 3 = 6$ Joule.

Άρα: $\Delta K = K_{\upsilon\sigma\tau\acute{\alpha}} - K_{\pi\rho\iota\nu} = 6 - 16 = -10$ J.

γ) $W_x = (m_1+m_2) g \eta\mu\varphi$



ΘΜΚΕ: Γ → Δ

$0 - \frac{1}{2}(m_1+m_2)V^2 = U_{\epsilon\lambda,\Gamma} - U_{\epsilon\lambda,\Delta} + W_{W_x(\Gamma\to\Delta)} \Rightarrow$

$-\frac{1}{2}(m_1+m_2)V^2 = \frac{1}{2}K\cdot\Delta\ell_1^2 - \frac{1}{2}K\cdot\Delta\ell_2^2 - (m_1+m_2)g\eta\mu\varphi\cdot(\Delta\ell_1-\Delta\ell_2)$

$\Rightarrow -\frac{1}{2}\cdot 4\cdot 3 = \frac{1}{2}\cdot 100\cdot 0{,}1^2 - \frac{1}{2}\cdot 100\cdot\Delta\ell_2^2 - 4\cdot 5\cdot(0{,}1-\Delta\ell_2) \Rightarrow$

$-12 = 4 - 100\Delta\ell_2^2 - 8 + 40\cdot\Delta\ell_2 \Rightarrow$

$100\Delta\ell_2^2 - 40\cdot\Delta\ell_2 - 8 = 0$

$\Delta = 1600 + 3200 = 4800$.

$\sqrt{\Delta} = 40\sqrt{3}$

$\Delta\ell_2 = \frac{40 \pm 40\sqrt{3}}{200} =$

$0{,}2 \pm 0{,}2\sqrt{3}$

$\Delta l_1 = 0{,}2\,m$

$\Delta l_2 < \Delta l_1$

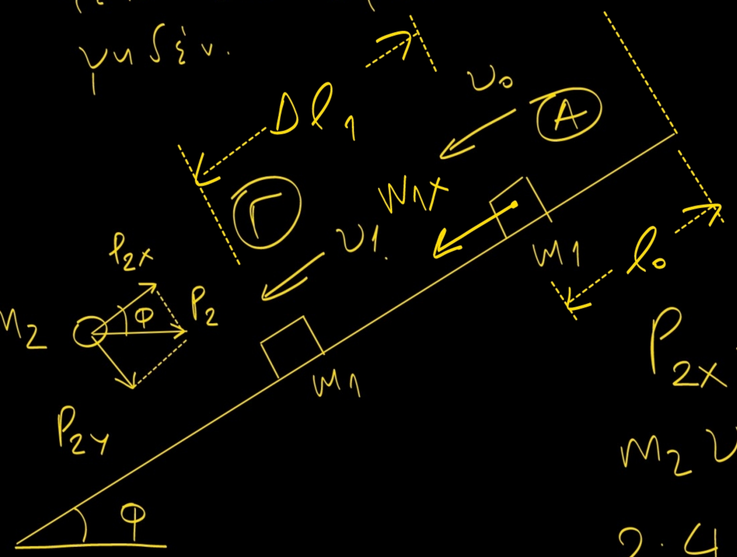
Άρα:

$\Delta l_2 = 0{,}2 - 0{,}2\sqrt{3}\ (m)$

οπότε:

$\Delta l_1 - \Delta l_2 = S = 0{,}2\sqrt{3}\ m$ το διανυόμενο διάστημα.

δ) Θέλουμε το συσσωμάτωμα μετά την κρούση να έχει ταχύτητα μηδέν.



$p_{2x} - m_1 v_1 = 0 \Rightarrow$

$m_2 v_2 \sigma\upsilon\nu\varphi - m_1 v_1 = 0 \Rightarrow$

$2 \cdot 4 \cdot \frac{\sqrt{3}}{2} = 2 v_1 \Rightarrow$

$v_1 = 2\sqrt{3}\ m/s.$

ΘΜΚΕ: Α → Γ

$K_\Gamma - K_A = W_{F_{\Sigma\alpha}(A\to\Gamma)} + W_{w_x(A\to\Gamma)} \Rightarrow$

$\frac{1}{2} m_1 v_1^2 - \frac{1}{2} m v_0^2 = 0 - \frac{1}{2} k \cdot \Delta l_1^2 + m_1 g \eta\mu\varphi \cdot \Delta l_1 \Rightarrow \frac{1}{2} \cdot 2 \cdot 4 \cdot 3 - \frac{1}{2} \cdot 2 \cdot v_0^2 = -\frac{1}{2} 100 \cdot 0{,}04 + 20 \cdot \frac{1}{2} \cdot 0{,}2 \Rightarrow$

$12 - v_0^2 = -2 + 2 \Rightarrow v_0^2 = 12 \Rightarrow v_0 = 2\sqrt{3}\ m/s.$