## Ασκ. 2.182 (Μαθ)

$$\left(v_2' = \frac{2m_1}{m_1+m_2}v_1 = \frac{4}{2+m_2}\cdot v_1\right)$$

α) $K_{2,\text{μετά}} = \frac{8}{9}K_{1,\text{πριν}}$ (1ος τρόπος)

ή

$K_2' = \frac{8}{9}K_1 \Rightarrow$

$\frac{1}{2}m_2 v_2'^2 = \frac{8}{9}\cdot\frac{1}{2}m_1 v_1^2 \Rightarrow$

$m_2 v_2'^2 = \frac{8}{9}m_1 v_1^2 \Rightarrow m_2\cdot\left(\frac{4}{2+m_2}\right)^2 \cancel{v_1^2} = \frac{8}{9}m_1\cancel{v_1^2} \Rightarrow$

$m_2\cdot\frac{16}{(2+m_2)^2} = \frac{16}{9} \Rightarrow 9m_2 = (2+m_2)^2 \Rightarrow$

$9m_2 = 4 + 4m_2 + m_2^2 \Rightarrow$

$m_2^2 - 5m_2 + 4 = 0$

$\Delta = 25 - 16 = 9$

$m_2 = \frac{5\pm 3}{2} \rightarrow \frac{8}{2} = 4\,\text{Kg}$

$\rightarrow \frac{2}{2} = 1\,\text{Kg}.$

Δε μεταβάλλεται η μάζα (δηλαδή η φορά) της $m_1$ δε μεγαλύτερη ελαστική κρούση, άρα $m_1 > m_2 \Rightarrow$

$m_2 = 1\,\text{Kg}.$

(2ος τρόπος)

$K_2' = \frac{P}{q} K_1$

Άρα: $K_1' = \frac{1}{9} K_1 \Rightarrow$

$\frac{1}{2} m_1 v_1'^2 = \frac{1}{9} \cdot \frac{1}{2} m_1 v_1^2 \Rightarrow$

$v_1'^2 = \frac{v_1^2}{9} \Rightarrow v_1' = \pm \frac{v_1}{3}$

Η $\Sigma_1$ δεν αλλάξει φορά

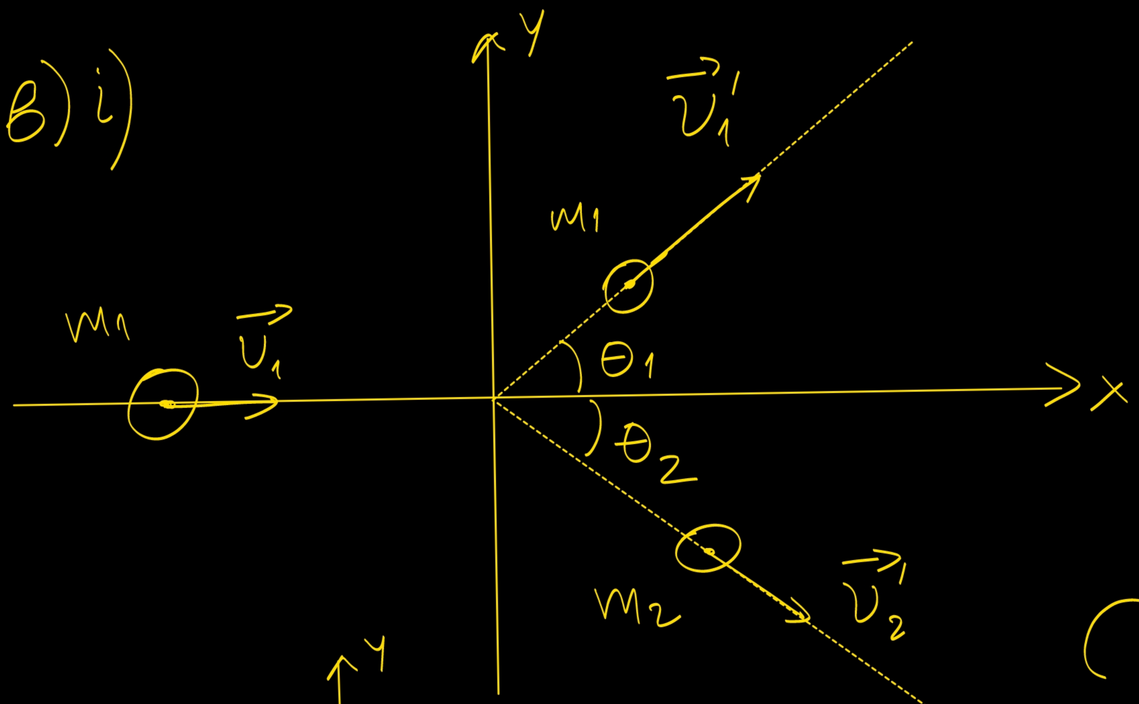
άρα: $v_1' = + \frac{v_1}{3} \Rightarrow \frac{m_1 - m_2}{m_1 + m_2} v_1 = \frac{1}{3} v_1 \Rightarrow \frac{m_1 - m_2}{m_1 + m_2} = \frac{1}{3} \Rightarrow$

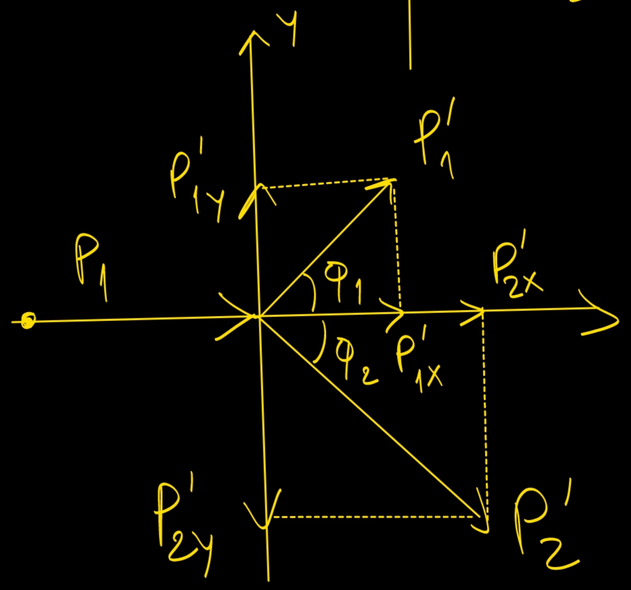
$3 m_1 - 3 m_2 = m_1 + m_2 \Rightarrow$

$2 m_1 = 4 m_2 \Rightarrow$

$4 = 4 m_2 \Rightarrow m_2 = 1 \text{ Kg}$

B) i)




$$p_{x,\text{πριν}} = p_{x,\text{μετά}} \Rightarrow$$

$$p_1 = p_1' \sigma\upsilon\nu\varphi_1 + p_2' \sigma\upsilon\nu\varphi_2 \quad (1)$$

$$p_{y,\text{πριν}} = p_{y,\text{μετά}} \Rightarrow$$

$$0 = p_{1y}' - p_{2y}' \Rightarrow 0 = p_1' \eta\mu\varphi_1 - p_2' \eta\mu\varphi_2 \quad (2)$$

$$(1): \; p_2' \sigma\upsilon\nu\varphi_2 = p_1 - p_1' \sigma\upsilon\nu\varphi_1 \;\Big|\Rightarrow$$

$$p_2' \cdot \eta\mu\varphi_2 = p_1' \eta\mu\varphi_1$$

$$p_2'^2 \sigma\upsilon\nu^2\varphi_2 + p_2'^2 \eta\mu^2\varphi_2 = (p_1 - p_1' \sigma\upsilon\nu\varphi_1)^2 + p_1'^2 \eta\mu^2\varphi_1 \Rightarrow$$

$$p_2'^2 (\sigma\upsilon\nu^2\varphi_2 + \eta\mu^2\varphi_2) = p_1^2 - 2 p_1 p_1' \sigma\upsilon\nu\varphi_1 + p_1'^2 \sigma\upsilon\nu^2\varphi_1 + p_1'^2 \eta\mu^2\varphi_1 \Rightarrow$$

$$p_2'^2 = p_1^2 - 2p_1p_1'\sigma\upsilon\nu\varphi_1 + p_1'^2\cdot(\sigma\upsilon\nu^2\varphi_1 + \eta\mu^2\varphi_1) \Rightarrow$$

$$p_2'^2 = p_1^2 - 2p_1p_1'\sigma\upsilon\nu\varphi_1 + p_1'^2$$

$$K = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$$

$$K = \frac{1}{2}mv\cdot v \Rightarrow \boxed{K = \frac{1}{2}p\cdot v}$$

$$\frac{p_1^2}{2m} = \frac{p_1'^2}{2m_1} + \frac{p_2'^2}{2m_2} \Rightarrow \frac{p_1^2 - p_1'^2}{m_1} = \frac{p_2'^2}{m_2} \Rightarrow p_2'^2 = \frac{m_2}{m_1}(p_1^2 - p_1'^2)$$

$$\frac{m_2}{m_1}(p_1^2 - p_1'^2) = p_1^2 - 2p_1p_1'\sigma\upsilon\nu\varphi_1 + p_1'^2 \Rightarrow$$

$$m_2p_1^2 - m_2p_1'^2 = m_1p_1^2 - 2m_1p_1p_1'\sigma\upsilon\nu\varphi_1 + m_1p_1'^2 \Rightarrow$$

$$(m_1+m_2)p_1'^2 - 2m_1p_1p_1'\sigma\upsilon\nu\varphi_1 + (m_1-m_2)p_1^2 = 0$$

Τριώνυμο ως προς $p_1'$ με

$\alpha = m_1+m_2$

$\beta = -2m_1p_1\sigma\upsilon\nu\varphi_1$ και $\gamma = +(m_1-m_2)p_1^2$