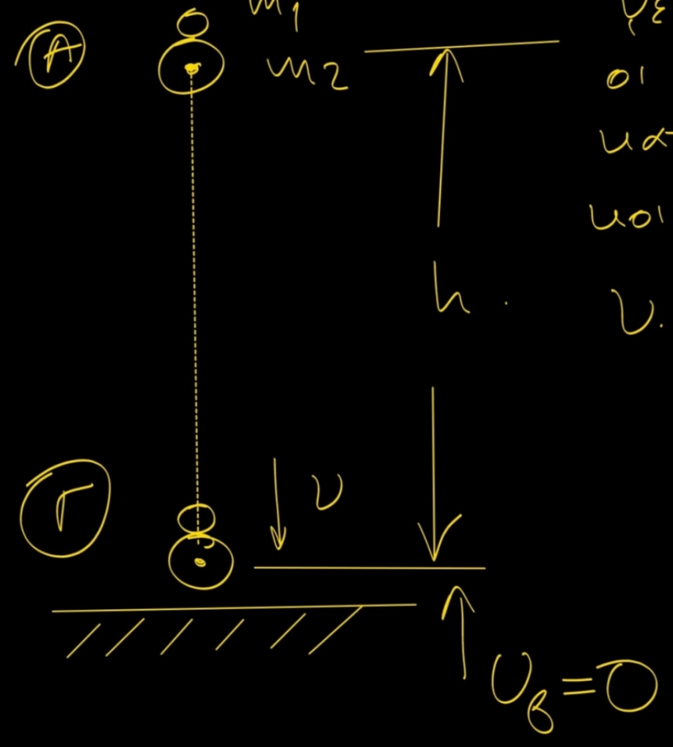
Άσκηση

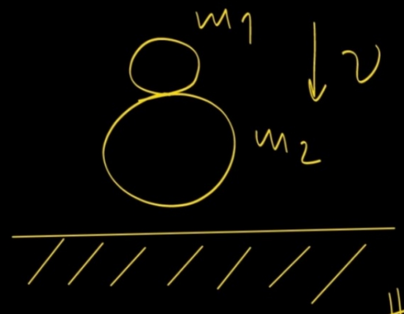


Πριν την κρούση με το έδαφος όλοι οι δύο υπάλξεις κατεβαίνουν με κοινή ταχύτητα v.

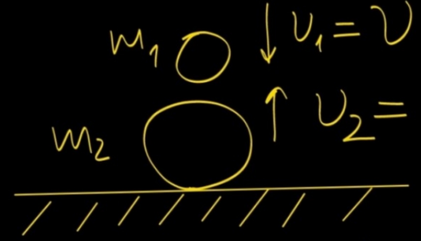
$$K_A+U_A=K_\Gamma+U_\Gamma \Rightarrow$$

$$0+m_1gh=\frac{1}{2}m_1v^2+0 \Rightarrow$$

$$v=\sqrt{2gh}.$$



(+) ↓



m_2 αναπηδάται ελαστικά στο έδαφος (διότι πολύ μεγάλης μάζας) και αμέσως μετά την $1^η$ κρούση ανεβαίνει με ταχύτητα $v_2=-v$ ενώ η m_1 ακόμα κατεβαίνει.

$$v_1' = \frac{m_1-m_2}{m_1+m_2}u_1 + \frac{2m_2}{m_1+m_2}u_2 = \frac{m_1-m_2}{m_1+m_2}\cdot v + \frac{2m_2}{m_1+m_2}(-v) \simeq$$

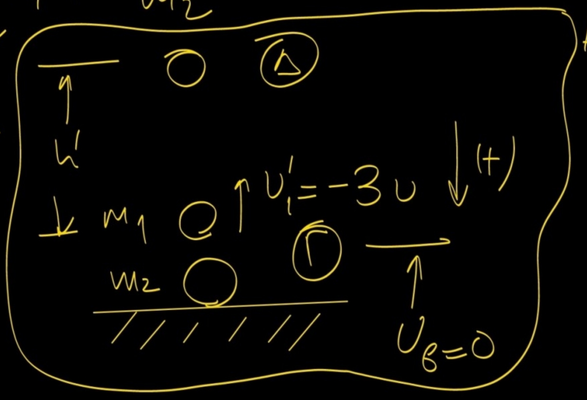
$$\simeq \frac{-m_2}{m_2}\cdot v + \frac{2m_2}{m_2}(-v) =$$

$$= -3v$$

$$v_2' = \frac{2m_1}{m_1+m_2}u_1 + \frac{m_2-m_1}{m_1+m_2}u_2$$

$$v_2' \simeq \frac{2m_1}{m_2}u_1 + \frac{m_2}{m_2}u_2 \Rightarrow$$

$$v_2' \simeq u_2$$



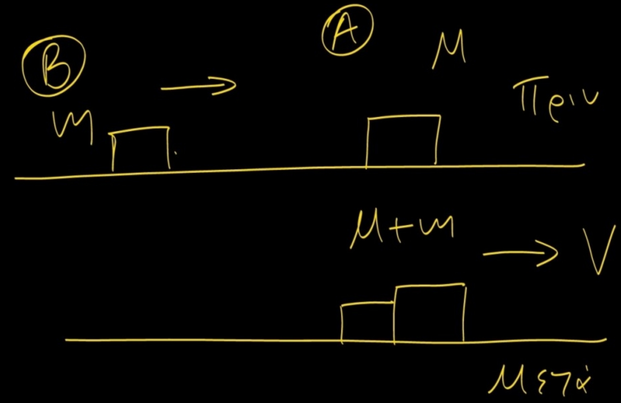
Για την m_1 μετά την πρόσκρουση έχουμε:

$$K_\Gamma + U_\Gamma = K_\Delta + U_\Delta \Rightarrow \frac{1}{2}m_1 v_1'^2 + 0 = m_1 g h' + 0 \Rightarrow$$

$$\frac{(-3v)^2}{2} = gh' \Rightarrow \frac{9v^2}{2} = gh' \Rightarrow \frac{9\cdot 2gh}{2} = gh'$$

$$\Rightarrow h' = 9h$$

Εφ. 1.38 | 6η Δ. 16.)



$P_{πριν} = P_{μετά} \Rightarrow$

$mU = (M+m)V$

$Q = \frac{1}{2}mU^2 - \frac{1}{2}(M+m)V^2 =$

$= \frac{1}{2}mU^2 - \frac{1}{2}(M+m)\frac{m^2U^2}{(M+m)^2} =$

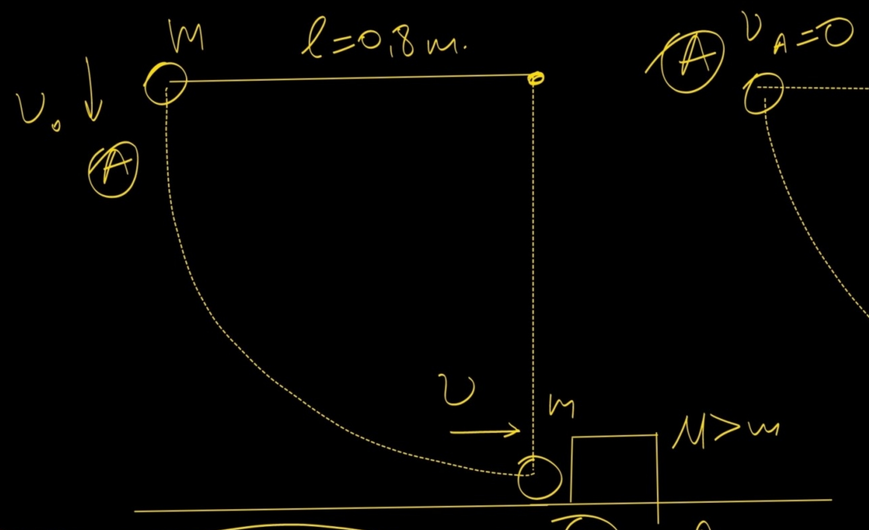
$= \frac{1}{2}mU^2\left(1 - \frac{m}{M+m}\right) \Rightarrow$

$Q = \frac{1}{2}mU^2 \cdot \frac{M}{M+m} \Rightarrow \frac{Q}{K_{αρχ}} = \frac{M}{M+m}$

$\frac{Q}{K_{αρχ}} \cdot 100\% = 80\% \Rightarrow Q = 0,8K_{αρχ}$

Άρα: $\frac{0,8K_{αρχ}}{K_{αρχ}} = \frac{M}{M+m} \Rightarrow 0,8M + 0,8m = M \Rightarrow M = 4m$

Αθ.1.56 (εξ.21.)



Αρα: $-4=\frac{m-M}{m+M}\cdot 8 \Rightarrow$

$+(m+M)=8m-8M \Rightarrow$

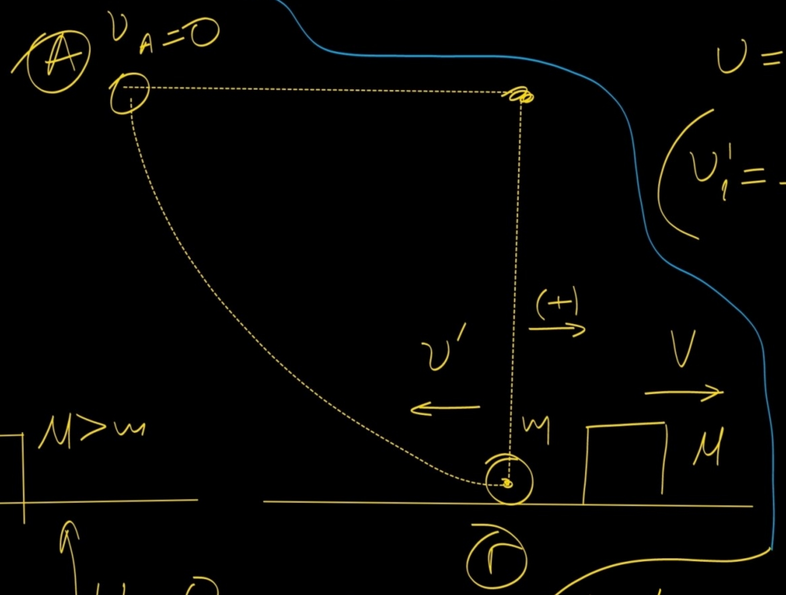
$m+M=2m-2M \Rightarrow m=3M \Rightarrow \frac{m}{M}=3$

α) $\frac{1}{2}mv_0^2+mg\ell=\frac{1}{2}mv^2 \Rightarrow v_0^2+2g\ell=v^2 \Rightarrow$

$16\cdot 3+2\cdot 0{,}8\cdot 10=v^2 \Rightarrow 16\cdot 4=v^2 \Rightarrow$

$v=8$ m/s.

$\left(v_1'=\frac{m_1-m_2}{m_1+m_2}v_1\right)$



$v'=\frac{m-M}{m+M}v$

ΑΔΕ: Γ → Α

$\frac{1}{2}mv'^2=mg\ell \Rightarrow$

$|v'|=\sqrt{2g\ell}=4$ m/s.

ή αν $v'=-4$ m/s

$$v' = \frac{m-M}{m+M} \cdot v \Rightarrow$$

$$-4 = \frac{m-M}{m+M} \cdot 8 \Rightarrow$$

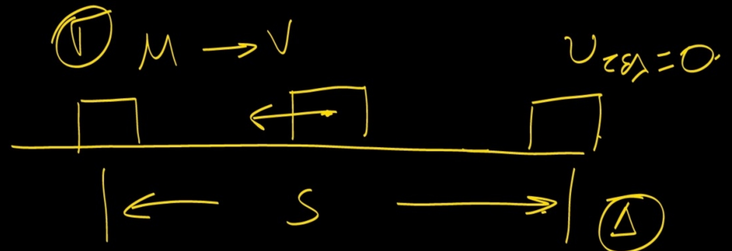
$$-m-M = 2m-2M \Rightarrow$$

$$+2M-M = 3m \Rightarrow$$

$$M = 3m \Rightarrow$$

$$\boxed{\frac{m}{M} = \frac{1}{3}}$$

β)



ΘΜΚΕ : Σ → Δ

$$0 - \frac{1}{2} M V^2 = -T \cdot S \Rightarrow$$

$$\frac{1}{2} M V^2 = -\mu M g S \Rightarrow$$

$$S = \frac{V^2}{2\mu g} = \frac{4^2}{2 \cdot 0{,}2 \cdot 10} = \frac{16}{4} = 4\,m.$$

γ)

$$\frac{p'_M}{p'_m} = \frac{MV}{-mv'} = \frac{3m \cdot 4}{-m \cdot 4} = -3$$

$$v'_2 = \frac{2m_1}{m_1+m_2} v_1$$

Αρα: $V = \frac{2m}{m+M} \cdot v =$

$$= \frac{2m}{m+3m} \cdot 8 =$$

$$= \frac{2}{4} \cdot 8 = 4\,m/s.$$

δ) $K_{πριν} = \frac{1}{2}mυ^2$

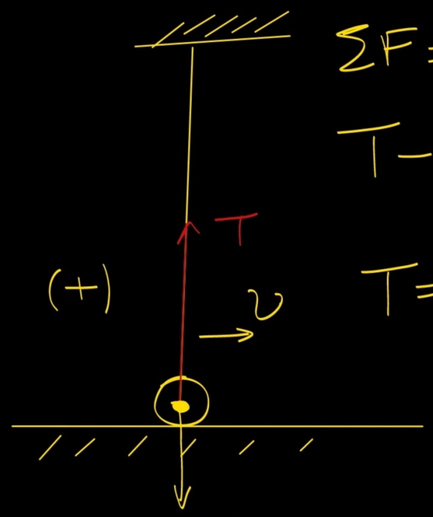
$K_{μετά} = \frac{1}{2}mυ'^2$

Ποσοστό μεταβολής $= \frac{ΔK}{K_{πριν}} \cdot 100 =$

$= \frac{K_{μετά} - K_{πριν}}{K_{πριν}} \cdot 100 = \frac{\frac{1}{2}mυ'^2 - \frac{1}{2}mυ^2}{\frac{1}{2}mυ^2} \cdot 100 =$

$= \frac{υ'^2 - υ^2}{υ^2} \cdot 100 = \frac{4^2 - 8^2}{4^2} \cdot 100 = \frac{-48}{64} \cdot 100 = -75\%$

ε)



$ΣF = F_K \Rightarrow$

$T - W = \frac{mυ^2}{ℓ} \Rightarrow$

$T = mg + \frac{mυ^2}{ℓ} =$

$= 10m + \frac{m \cdot 8^2}{0,8}$

$= 10m + 80m$

$= 90m$

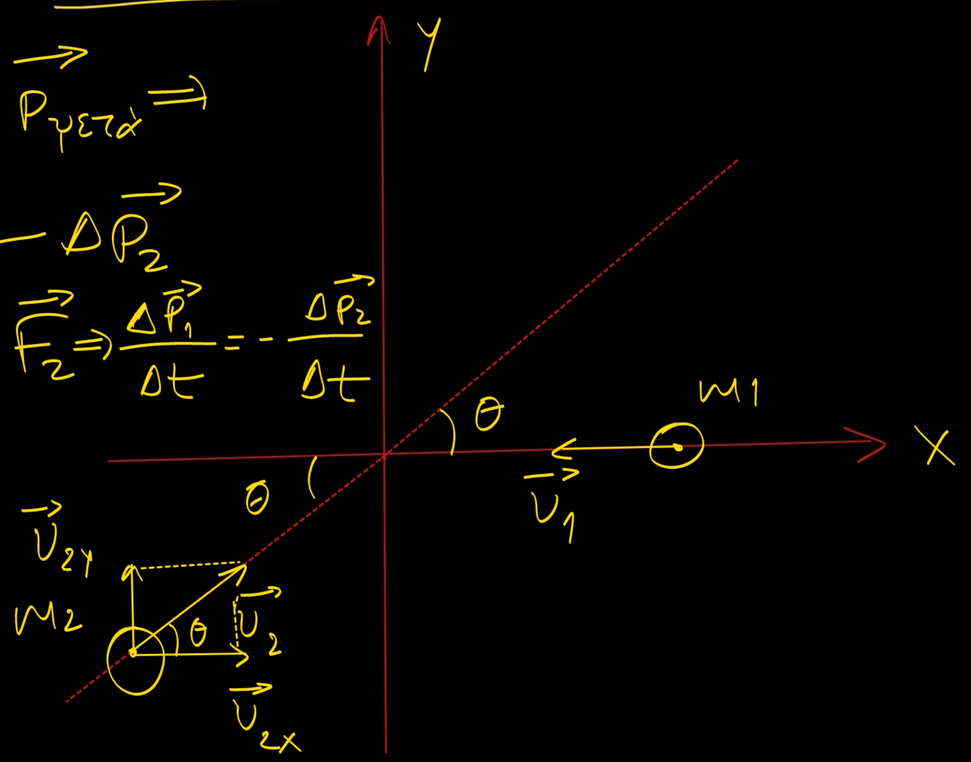
Ομοίως:

$T' = mg + \frac{mυ'^2}{ℓ} =$

$T' = 10m + \frac{m \cdot 16}{0,8} = 30m$

Άρα: $\frac{T}{T'} = \frac{90m}{30m} = 3$

Άλυ. 1.50 σελ. 18)



$\vec{P}_{\pi\rho\iota\nu} = \vec{P}_{\upsilon\sigma\tau\acute{\alpha}\tau} \Rightarrow$

$\vec{\Delta P}_1 = -\vec{\Delta P}_2$

ή $\vec{F}_1 = -\vec{F}_2 \Rightarrow \frac{\vec{\Delta P}_1}{\Delta t} = -\frac{\vec{\Delta P}_2}{\Delta t}$

$P_{x,\pi\rho\iota\nu} = m_2 v_{2x} - m_1 v_1.$

$P_{y,\pi\rho\iota\nu} = m_2 v_{2y}.$

$P_{x,\upsilon\sigma\tau\acute{\alpha}\tau} = P_{x,\pi\rho\iota\nu} = m_2 v_2 \cdot \sigma\upsilon\nu\theta - m_1 v_1 =$

$= m \cdot 5v \cdot 0{,}6 - m \cdot v =$

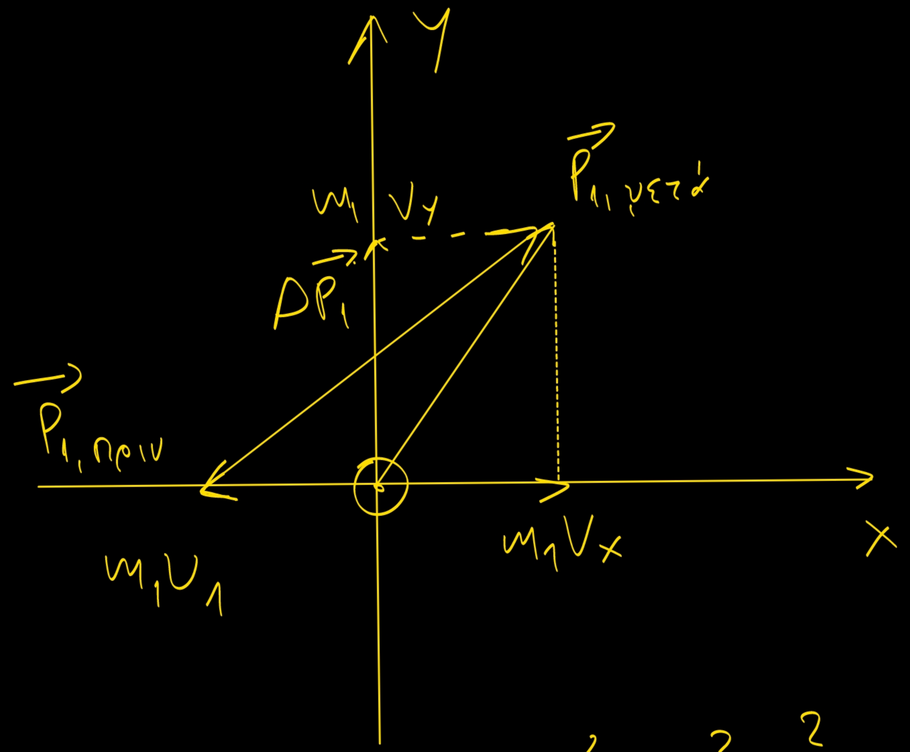
$= 3mv - mv = 2mv.$

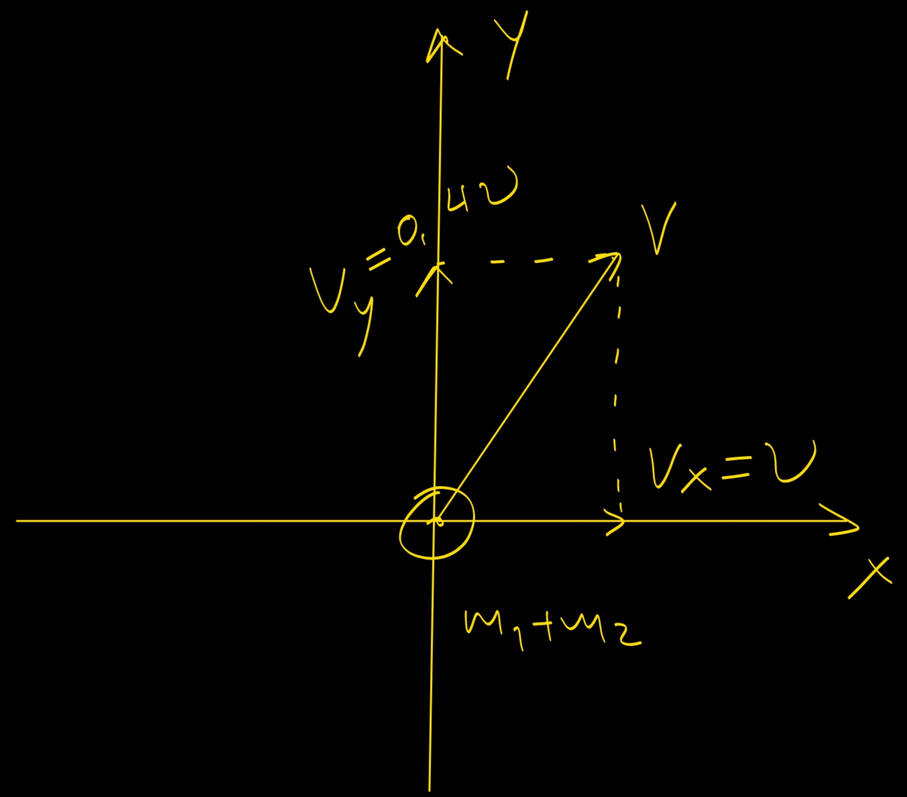
$P_{y,\upsilon\sigma\tau\acute{\alpha}\tau} = P_{y,\pi\rho\iota\nu} = m_2 v_{2y} = m_2 v_2 \cdot \eta\mu\theta =$

$= 5m \cdot v \cdot 0{,}8 = 4mv.$

$MV_x = 2mv \Rightarrow 2mV_x = 2mv \Rightarrow V_x = v$

$MV_y = 4mv \Rightarrow 2mV_y = 4mv \Rightarrow V_y = 2v$





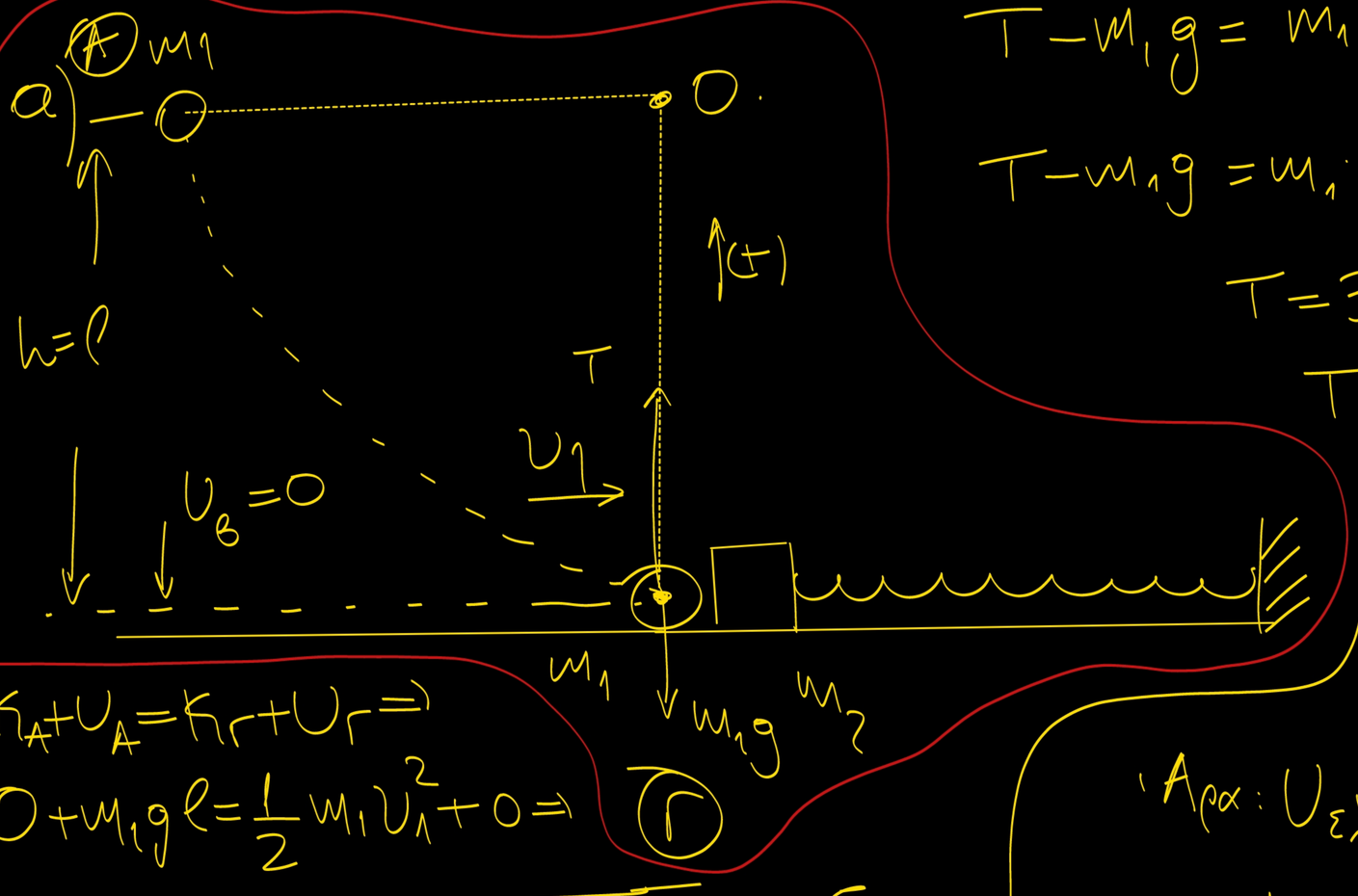
$$\Delta p_1^2 = (m_1 u_1 + m_1 v_x)^2 + m_1^2 v_y^2 =$$

$$= (mv + mv)^2 + (m \cdot 2v)^2 = 4m^2v^2 + 4m^2v^2 \Rightarrow \Delta p_1^2 = 8m^2v^2 \Rightarrow$$

$$\Delta p_1 = 2\sqrt{2}\, mv$$

Σωστό το (β)

Ασκ. 1.59) Θεμ. 23



$K_A+U_A=K_B+U_B \Rightarrow$

$0+m_1 g\ell=\frac{1}{2}m_1 U_1^2+0 \Rightarrow$

$U_1=\sqrt{2g\ell}=\sqrt{2\cdot 10\cdot 1,25}=5\ m/s.$

$\Sigma F_{(κεντρικά)}=F_κ \Rightarrow$

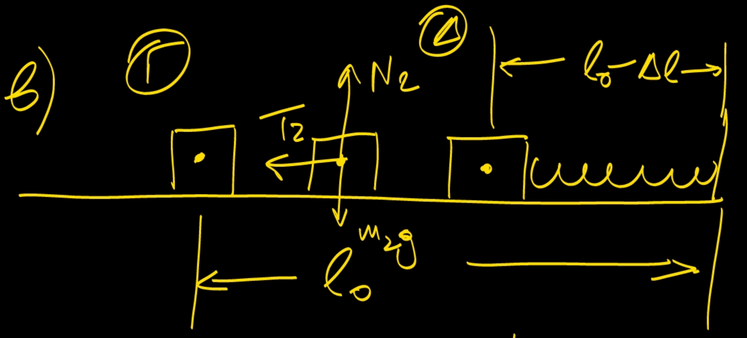
$T-m_1 g=m_1\frac{U_1^2}{\ell}\Rightarrow$

$T-m_1 g=m_1\cdot\frac{2g\ell}{\ell}\Rightarrow$

$T=3m_1 g \Rightarrow$

$T=3\cdot 2\cdot 10=60N$

β)



$Q=|W_{T_2}|=$

$=|-T_2\cdot\Delta\ell|=$

$=|-\mu N_2\cdot\Delta\ell|\Rightarrow$

$Q=\mu m_2 g\cdot\Delta\ell\Rightarrow$

$15=0,5\cdot 3\cdot 10\cdot\Delta\ell\Rightarrow$

$15=15\Delta\ell\Rightarrow$

$\Delta\ell=1\ m.$

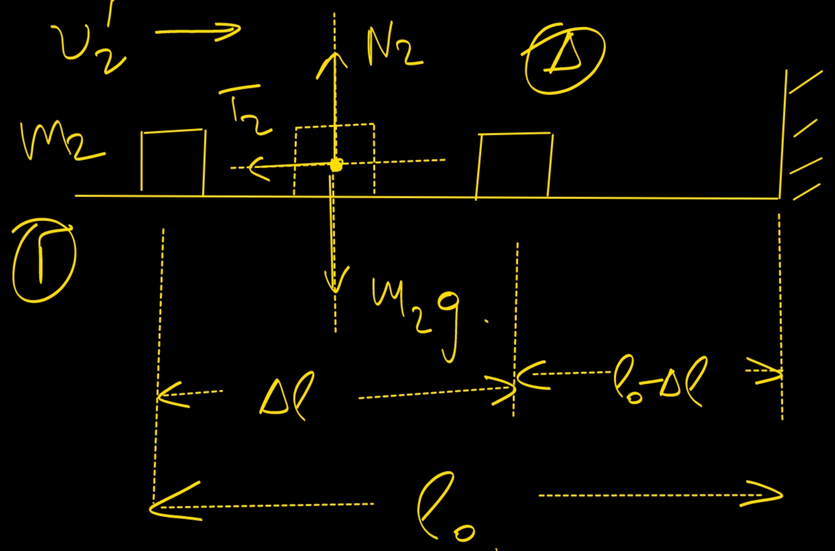
$A_{ελ}: U_{ελ,max}=\frac{1}{2}k\cdot\Delta\ell^2=$

$=\frac{1}{2}\cdot 18\cdot 1^2=9\ Joule.$

γ) Πριν την κρούση έχουμε:

$U_1 = 5\ m/s$.

και $U_2 = 0\ m/s$.



Αν ήταν ελαστική θα

ίσχυε: $U_2' = \frac{2m_1}{m_1+m_2} U_1 = \frac{2 \cdot 2}{5} \cdot 5 = 4\ m/s$.

Άρα είναι ελαστική.

$$W_{F_{\epsilon\lambda}(\Gamma \to \Delta)} = U_{\epsilon\lambda(\Gamma)} - U_{\epsilon\lambda(\Delta)} = \frac{1}{2} K \cdot 0^2 - \frac{1}{2} \cdot K \cdot \Delta l^2 =$$

$$= -\frac{1}{2} K \cdot \Delta l^2 .$$

$$W_T(\Gamma \to \Delta) = -T_2 \cdot \Delta l = -15\ Joule.$$

και $W_{F_{\epsilon\lambda}} = -9\ Joule$.

ΘΜΚΕ: $\Gamma \to \Delta$: $\Delta K = \Sigma W \Rightarrow$

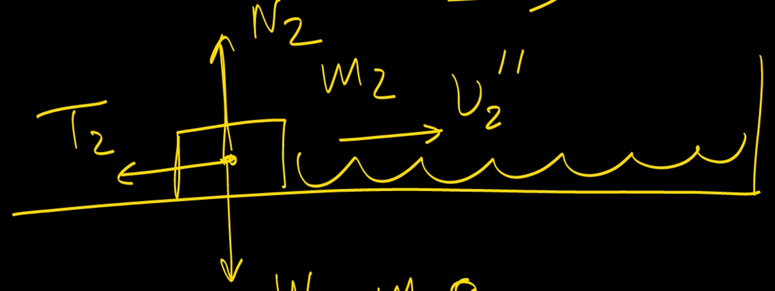
$$0 - \frac{1}{2} m_2 U_2'^2 = W_{F_{\epsilon\lambda}} + W_T \Rightarrow -\frac{1}{2} \cdot 3 \cdot U_2'^2 = -24 \Rightarrow$$

$$U_2'^2 = \frac{48}{3} = 16 \Rightarrow U_2' = 4\ m/s .$$

δ) Πάλι έχουμε: $U_1 = 5\,m/s$.

$$U_2'' = \frac{2m_1'}{m_1' + m_2} U_1 = \frac{2 \cdot 1}{1+3} \cdot 5 = 2{,}5\,m/s.$$

$(+) \rightarrow$



$$K_{1\,πριν} = \frac{1}{2} m_1 U_1^2 = \frac{1}{2} \cdot 1 \cdot 25 = 12{,}5\,Joule.$$

$$K_{2,\,μετά} = \frac{1}{2} m_2 U_2''^2 = \frac{1}{2} \cdot 3 \cdot 2{,}5^2 = \frac{3 \cdot 12{,}5}{4}$$

$U = σταθερή = mgh = 0\,Joule$

Άρα: $\frac{\Delta U}{\Delta t} = 0$.

Άρα: Ποσοστό $= \frac{K_{2,\,μετά}}{K_{1\,πριν}} \cdot 100 = \frac{\frac{3 \cdot 12{,}5}{4}}{12{,}5} \cdot 100 = 75\%$

$$\frac{\Delta K}{\Delta t} = \frac{\Sigma \vec{F} \cdot \Delta \vec{x}}{\Delta t} = \Sigma \vec{F} \cdot \frac{\Delta \vec{x}}{\Delta t} = \Sigma \vec{F} \cdot \vec{U}$$

$E = K + U \Rightarrow$

$$\frac{\Delta E}{\Delta t} = \frac{\Delta K}{\Delta t} + \frac{\Delta U}{\Delta t} = + \Sigma F_x \cdot U_2'' = - T_2 \cdot U_2'' = - \mu m_2 g \cdot U_2'' = -0{,}5 \cdot 3 \cdot 10 \cdot 2{,}5 =$$

$$= -37{,}5\,Joule/sec.$$

$F_{εξ} = 0$ $\quad \Sigma F_x = -T_2$