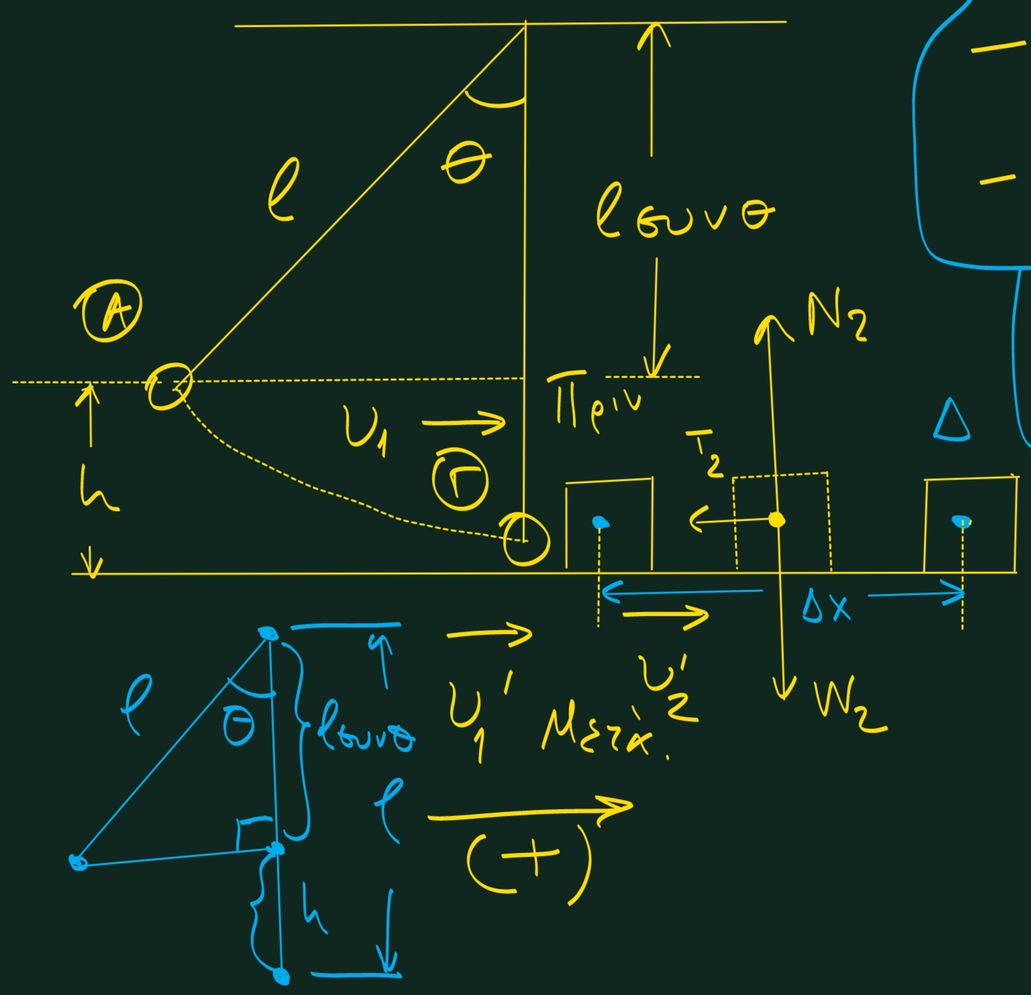
Ασκ. 1.117 (Σαββ)



α) $\Delta P_1 = m_1 U_1' - m_1 U_1 \Rightarrow$

$-12 = 2U_1' - 2U_1 \Rightarrow$

$-6 = U_1' - U_1$

$U_1' = \frac{m_1 - m_2}{m_1 + m_2} U_1 \Rightarrow$

$U_1' = \frac{2-3}{2+3} U_1 \Rightarrow$

$U_1' = -\frac{U_1}{5}$

$T_2 = \mu N_2 = \mu m_2 g$

$-6 = U_1' - U_1 \Rightarrow$

$-6 = -\frac{U_1}{5} - U_1 \Rightarrow$

$-6 = -\frac{6U_1}{5} \Rightarrow U_1 = 5\,m/s.$

$U_2' = \frac{2m_1}{m_1 + m_2} U_1 = \frac{2 \cdot 2}{2+3} \cdot 5 \Rightarrow$

$U_2' = 4\,m/s.$

Παίρνουμε ΘΜΚΕ για το m_2 μετά την κρούση:

ΘΜΚΕ: Γ → Δ:

$\Delta K = \Sigma W \Rightarrow 0 - \frac{1}{2} m_2 U_2'^2 = -T_2 \cdot \Delta x \Rightarrow$

$-\frac{1}{2} m_2 U_2'^2 = -\mu m_2 g \Delta x \Rightarrow$

$\Delta x = \frac{U_2'^2}{2\mu g} \Rightarrow \Delta x = \frac{16}{2 \cdot 0{,}1 \cdot 10} \Rightarrow$

$\Delta x = 8\,m$

β) ΑΔΕ για το m_1 από το Α στο Γ:

$$m_1 g h = \frac{1}{2} m_1 v_1^2 \Rightarrow$$

$$h = \frac{v_1^2}{2g} \Rightarrow h = \frac{5^2}{2 \cdot 10} = \frac{25}{20} = 1{,}25\,m.$$

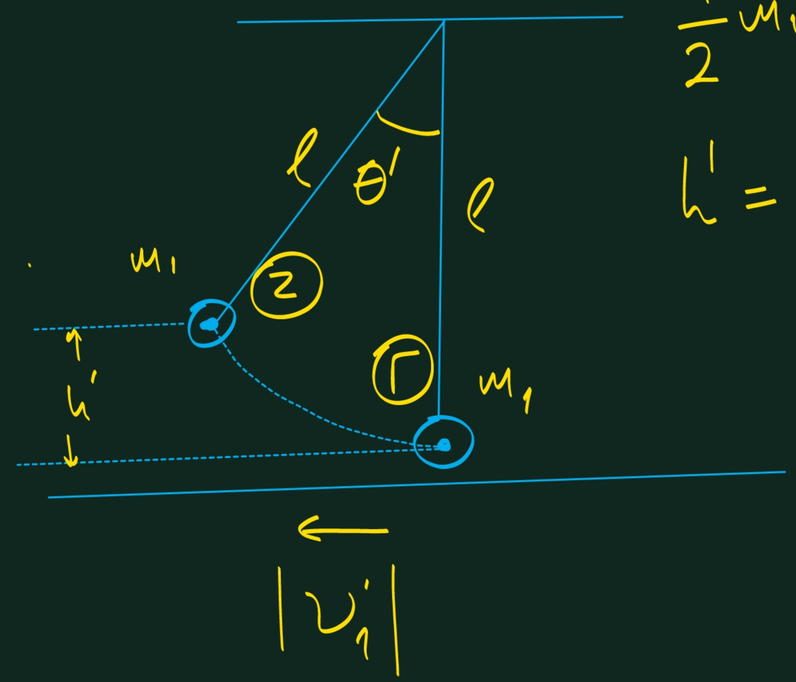
Από το σχήμα έχουμε: $h + \ell \cdot \sigma\upsilon\nu\theta = \ell \Rightarrow$

$$1{,}25 + 2{,}5 \cdot \sigma\upsilon\nu\theta = 2{,}5 \Rightarrow$$

$$2{,}5\,\sigma\upsilon\nu\theta = 1{,}25 \Rightarrow \sigma\upsilon\nu\theta = \frac{1}{2} \Rightarrow$$

$$\theta = 60^\circ$$

γ) $v_1' = -\frac{v_1}{5}$



ΑΔΕ: Γ → Ζ

$$\frac{1}{2} m_1 v_1'^2 = m_1 g h' \Rightarrow$$

$$h' = \frac{v_1'^2}{2g} = \frac{1}{20} \Rightarrow$$

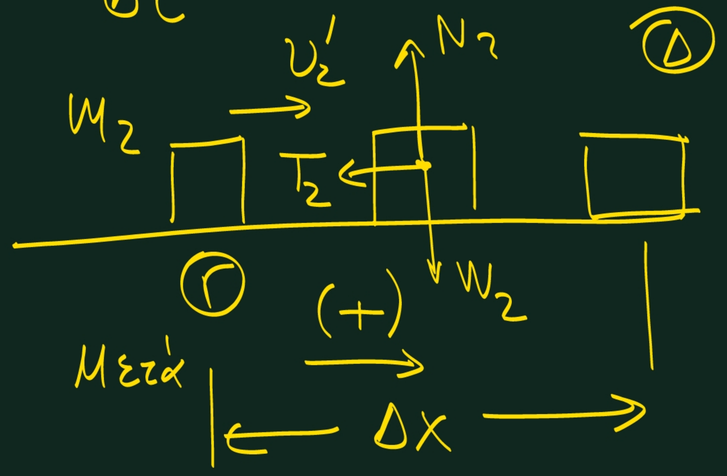
$$h' = 0{,}05\,m$$

$$v_1' = -\frac{5}{5} = -1\,m/s.$$

δ) Ο ρυθμός μεταβολής της ορμής είναι η συνισταμένη δύναμη.

$$\frac{\Delta \vec{p}}{\Delta t} = \Sigma \vec{F}$$ ή απλά:

$$\frac{\Delta p}{\Delta t} = \Sigma F$$



$$\Sigma \vec{F} = \Sigma \vec{F}_x + \Sigma \vec{F}_y$$

$$\Sigma F_y = N_2 - w_2 = 0$$

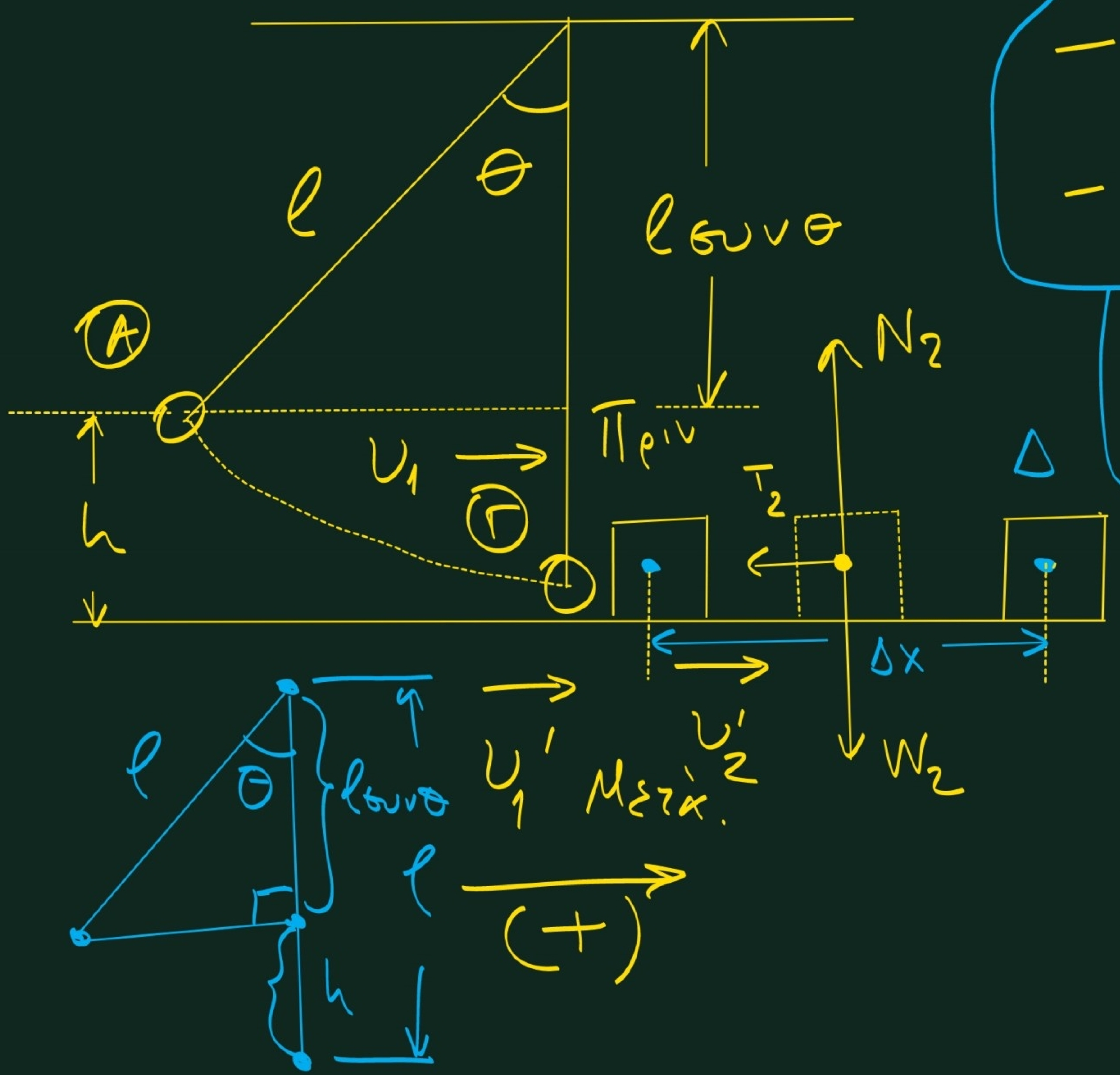
Άρα: $\Sigma F = \Sigma F_x =$

$$\Sigma F = -T_2 = -\mu N_2 = -\mu m_2 g$$

Συνεπώς: $\frac{\Delta p}{\Delta t} = -T_2 = -\mu m_2 g =$

$$\frac{\Delta p}{\Delta t} = -0{,}1 \cdot 3 \cdot 10 \Rightarrow \frac{\Delta p}{\Delta t} = -3\ kg \cdot m/s^2$$

Ασκ. 1.117 (Σαββ)



α) $\Delta P_1 = m_1 υ_1' - m_1 υ_1 \Rightarrow$

$-12 = 2υ_1' - 2υ_1 \Rightarrow$

$-6 = υ_1' - υ_1$

$-6 = υ_1' - υ_1 \Rightarrow$

$-6 = -\frac{υ_1}{5} - υ_1 \Rightarrow$

$-6 = -\frac{6υ_1}{5} \Rightarrow υ_1 = 5\,m/s.$

$υ_1' = \frac{m_1 - m_2}{m_1 + m_2} υ_1 \Rightarrow$

$υ_1' = \frac{2-3}{2+3} υ_1 \Rightarrow$

$υ_1' = -\frac{υ_1}{5}$

$υ_2' = \frac{2m_1}{m_1 + m_2} υ_1 = \frac{2 \cdot 2}{2+3} \cdot 5 \Rightarrow$

$υ_2' = 4\,m/s.$

Παίρνουμε ΘΜΚΕ για το m_2 μετά την κρούση:

ΘΜΚΕ: (Γ) → (Δ):

$\Delta K = \Sigma W \Rightarrow 0 - \frac{1}{2} m_2 υ_2'^2 = -T_2 \cdot \Delta x \Rightarrow$

$-\frac{1}{2} m_2 υ_2'^2 = -μ m_2 g \Delta x \Rightarrow$

$T_2 = μ N_2 = μ m_2 g$

$\Delta x = \frac{υ_2'^2}{2μg} \Rightarrow \Delta x = \frac{16}{2 \cdot 0{,}1 \cdot 10} \Rightarrow$ $\Delta x = 8\,m$

β) ΑΔΕ για το m_1 από το Α στο Γ:

$$m_1 g h = \frac{1}{2} m_1 v_1^2 \Rightarrow$$

$$h = \frac{v_1^2}{2g} \Rightarrow h = \frac{5^2}{2 \cdot 10} = \frac{25}{20} = 1{,}25\,m.$$

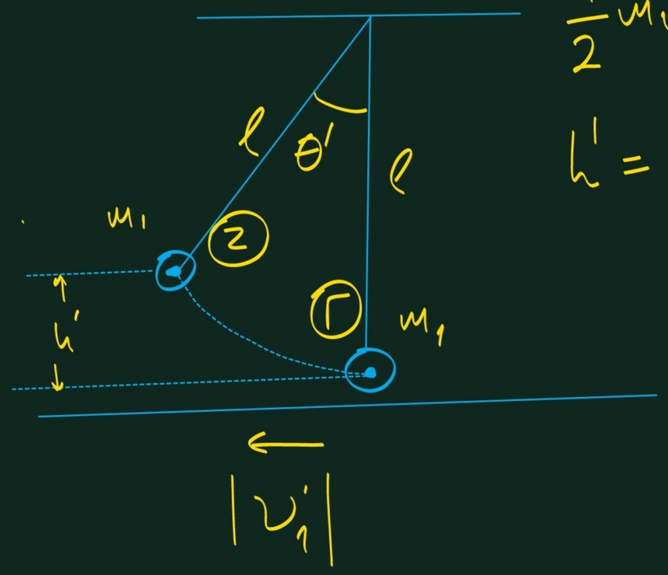
Από το σχήμα έχουμε: $h + \ell \cdot \sigma\upsilon\nu\theta = \ell \Rightarrow$

$$1{,}25 + 2{,}5 \cdot \sigma\upsilon\nu\theta = 2{,}5 \Rightarrow$$

$$2{,}5\,\sigma\upsilon\nu\theta = 1{,}25 \Rightarrow \sigma\upsilon\nu\theta = \frac{1}{2} \Rightarrow$$

$$\theta = 60^\circ$$

γ) $v_1' = -\frac{v_1}{5}$



$$\leftarrow$$

$$|v_1'|$$

$$v_1' = -\frac{5}{5} = -1\,m/s.$$

ΑΔΕ: Γ → Δ

$$\frac{1}{2} m_1 v_1'^2 = m_1 g h' \Rightarrow$$

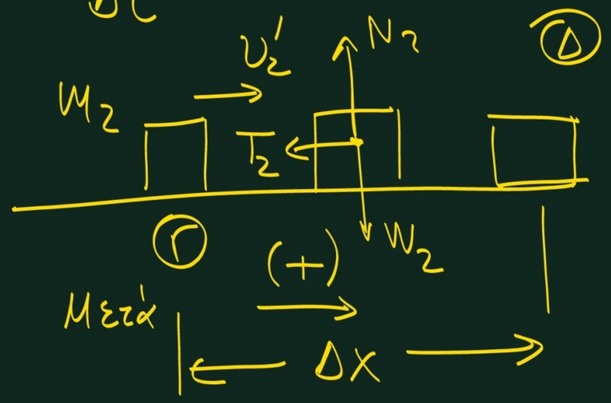
$$h' = \frac{v_1'^2}{2g} = \frac{1}{20} \Rightarrow$$

$$h' = 0{,}05\,m$$

δ) Ο ρυθμός μεταβολής της ορμής είναι η συνισταμένη δύναμη.

$$\frac{\Delta \vec{p}}{\Delta t} = \Sigma \vec{F}$$ ή απλά:

$$\frac{\Delta p}{\Delta t} = \Sigma F$$



$$\Sigma \vec{F} = \Sigma \vec{F}_x + \Sigma \vec{F}_y$$

$$\Sigma F_y = N_2 - W_2 = 0$$

Άρα: $\Sigma F = \Sigma F_x \Rightarrow$

$$\Sigma F = -T_2 = -\mu N_2 = -\mu m_2 g$$

Συνεπώς: $\frac{\Delta p}{\Delta t} = -T_2 = -\mu m_2 g \Rightarrow$

$$\frac{\Delta p}{\Delta t} = -0,1 \cdot 3 \cdot 10 \Rightarrow \frac{\Delta p}{\Delta t} = -3\ kg \cdot m/s^2$$

Ρυθμός μεταβολής κινητικής ενέργειας.

$$K = \frac{1}{2} m v^2$$

$$K(t) = \frac{1}{2} m (v(t))^2$$

Ο απλούστερος τρόπος εύρεσης του $\frac{\Delta K}{\Delta t}$ είναι:

$$\frac{\Delta K}{\Delta t} = \frac{\Sigma F \cdot \Delta x}{\Delta t} = \Sigma F \cdot \frac{\Delta x}{\Delta t} = \Sigma F \cdot v$$

$$\left(\text{ή } \frac{\Delta K}{\Delta t} = \Sigma \vec{F} \cdot \vec{v} \right)$$

Δηλαδή $\frac{\Delta K}{\Delta t} = \pm \Sigma F \cdot v$ ανάλογα με το πρόσημο.

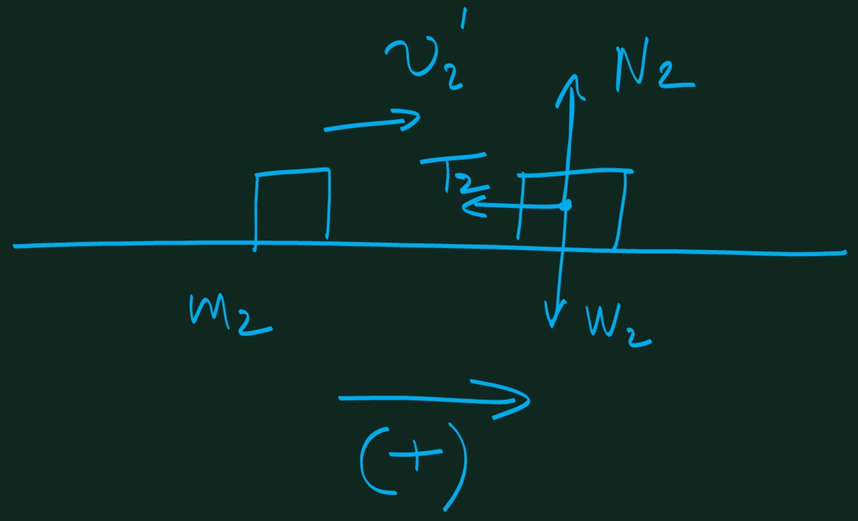
$$K = \frac{1}{2} m v^2 \Rightarrow$$

$$\frac{dK}{dt} = \frac{1}{2} m \frac{d}{dt}(v^2) \Rightarrow$$

$$\frac{dK}{dt} = \frac{1}{2} \cdot m \cdot 2v \cdot \frac{dv}{dt} \Rightarrow$$

$$\frac{dK}{dt} = mv \frac{dv}{dt} \Rightarrow$$

$$\frac{dK}{dt} = ma v = \Sigma F \cdot v$$



$$T_2 = \mu N_2 = \mu m_2 g = 0,1 \cdot 3 \cdot 10 = 3\,N$$

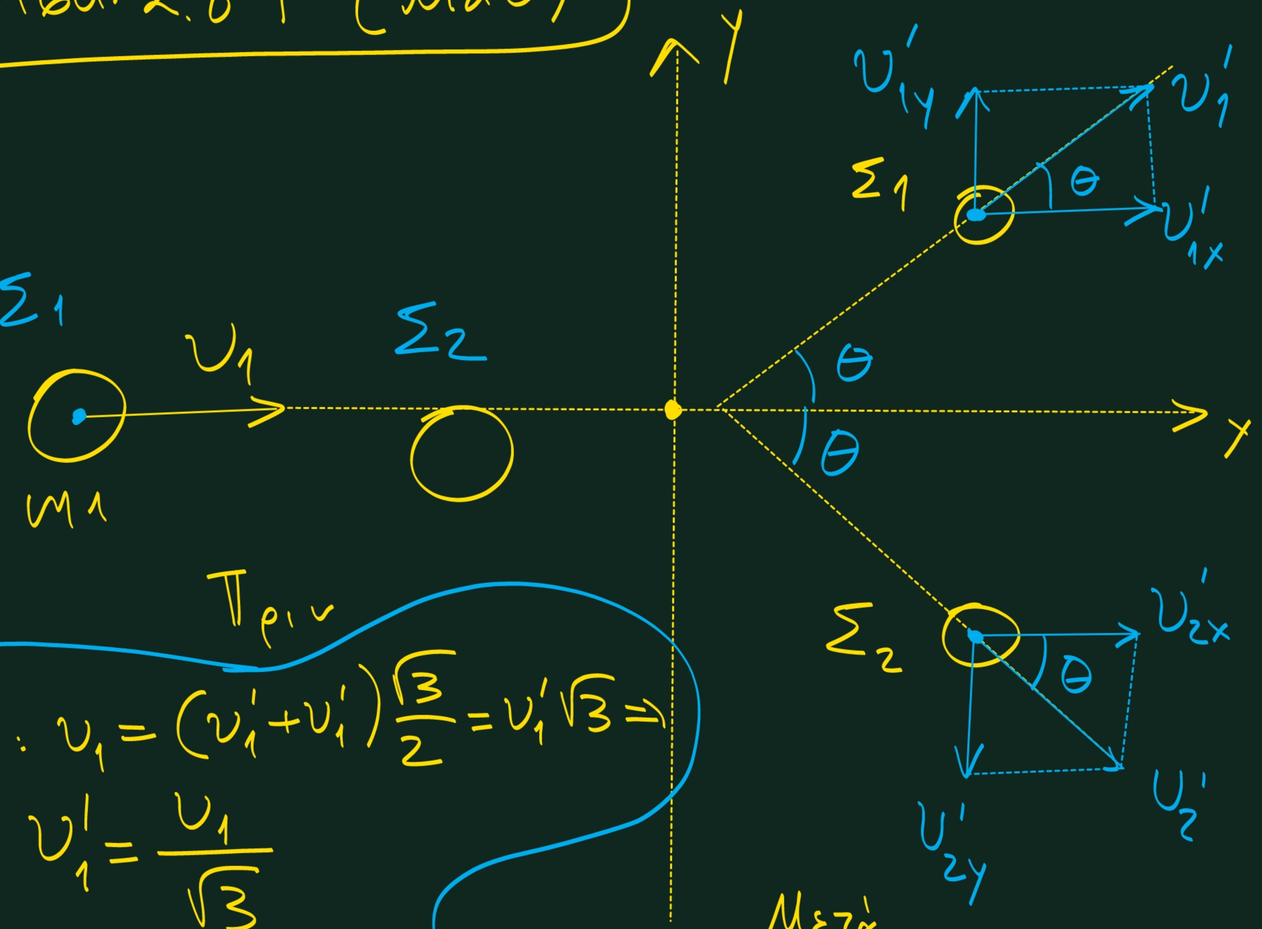
$$\upsilon_2' = 4\,m/s$$

$$\Sigma \vec{F} = \Sigma \vec{F}_x + \Sigma \vec{F}_y = \vec{0} = \Sigma \vec{F}_x$$

$$\Sigma F = -T_2$$

$$\text{Αρα: } \frac{\Delta K}{\Delta t} = \Sigma F \cdot \upsilon = -T_2 \cdot \upsilon_2' \Rightarrow \frac{\Delta K}{\Delta t} = -3 \cdot 4 \Rightarrow \boxed{\frac{\Delta K}{\Delta t} = -12\ J/sec}$$

Ασκ. 2.87 (Μαθ.)



Πριν

Αρα: $v_1 = (v_1' + v_1')\frac{\sqrt{3}}{2} = v_1'\sqrt{3} \Rightarrow$

$v_1' = \frac{v_1}{\sqrt{3}}$

και $v_2' = \frac{v_1}{3\sqrt{3}}$

$m_2 = 3m_1$.

$p_{x,\text{πριν}} = p_{x,\text{μετά}} \Rightarrow$

$m_1 v_1 = m_1 v_1' \sigma\upsilon\nu\theta + m_2 v_2' \sigma\upsilon\nu\theta \Rightarrow$

$m_1 v_1 = m_1 v_1' \sigma\upsilon\nu\theta + 3 m_1 v_2' \sigma\upsilon\nu\theta \Rightarrow$

$v_1 = \left(v_1' + 3v_2'\right) \cdot \sigma\upsilon\nu 30^\circ \Rightarrow$

$v_1 = \left(v_1' + 3v_2'\right) \cdot \frac{\sqrt{3}}{2}$

$p_{y,\text{πριν}} = p_{y,\text{μετά}} \Rightarrow$

$0 = m_1 v_{1y}' - m_2 v_{2y}' \Rightarrow$

$\cancel{m_1} v_1' \cancel{\eta\mu\theta} = 3\cancel{m_1} v_2' \cancel{\eta\mu\theta} \Rightarrow v_1' = 3v_2'$

$$K = \frac{1}{2} m_1 v_1^2$$

$$K_1' = \frac{1}{2} m_1 v_1'^2 = \frac{1}{2} m_1 \cdot \frac{v_1^2}{3} = \frac{1}{3} \cdot \frac{1}{2} m_1 v_1^2 = \frac{K}{3}$$

$$K_2' = \frac{1}{2} m_2 v_2'^2 = \frac{1}{2} \cdot 3 m_1 \cdot \frac{v_1^2}{9 \cdot 3} = \frac{1}{9} \cdot \frac{1}{2} m_1 v_1^2 = \frac{K}{9}$$

$$v_1' = \frac{v_1}{\sqrt{3}}$$

$$|\Delta K| = K - K_1' - K_2' = K - \frac{K}{3} - \frac{K}{9} = K - \frac{4K}{9} \Rightarrow$$

$$v_2' = \frac{v_1}{3\sqrt{3}}$$

$$|\Delta K| = \frac{5K}{9}$$ Σωστό το (Β).