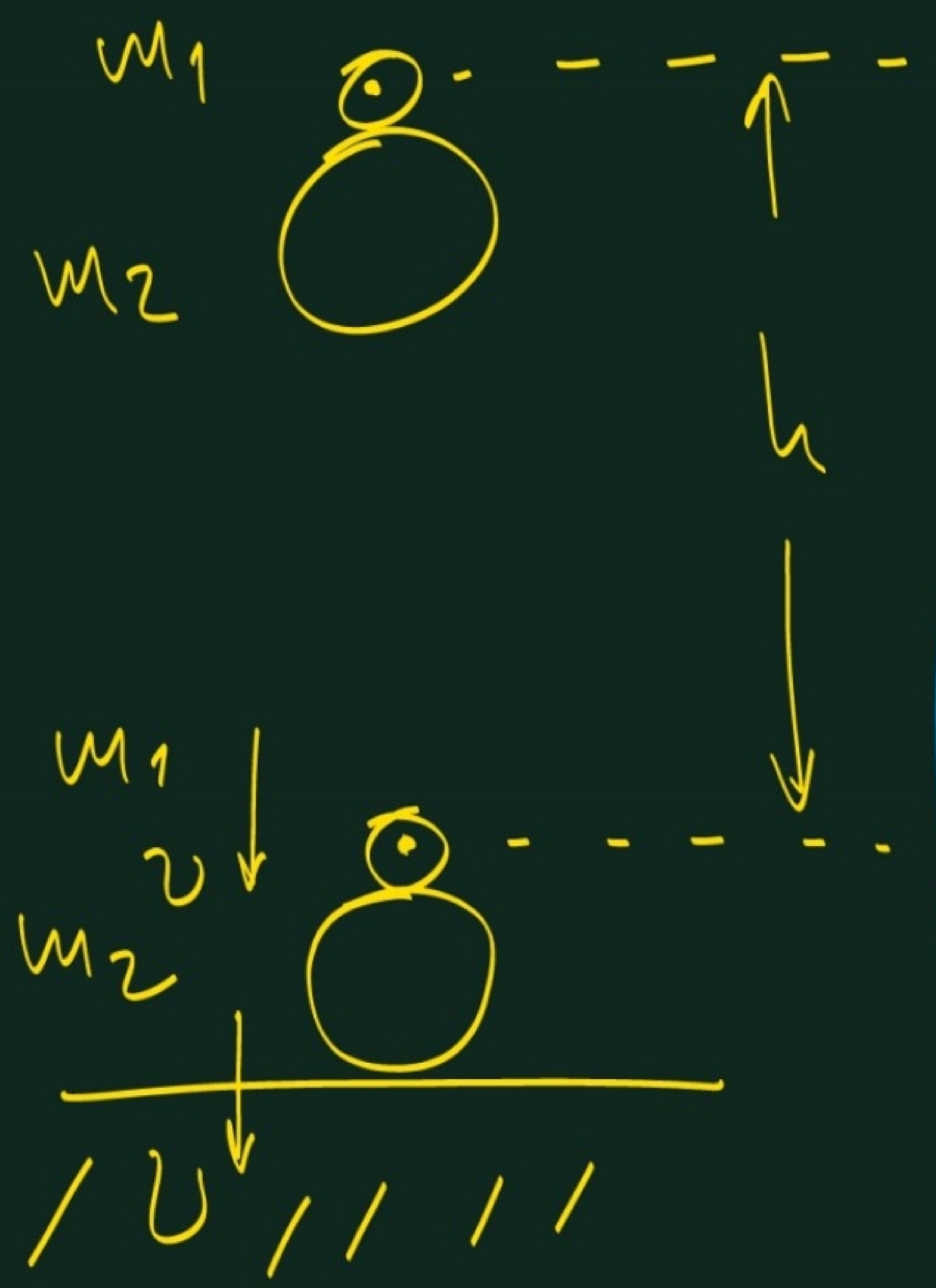
Άσκηση.

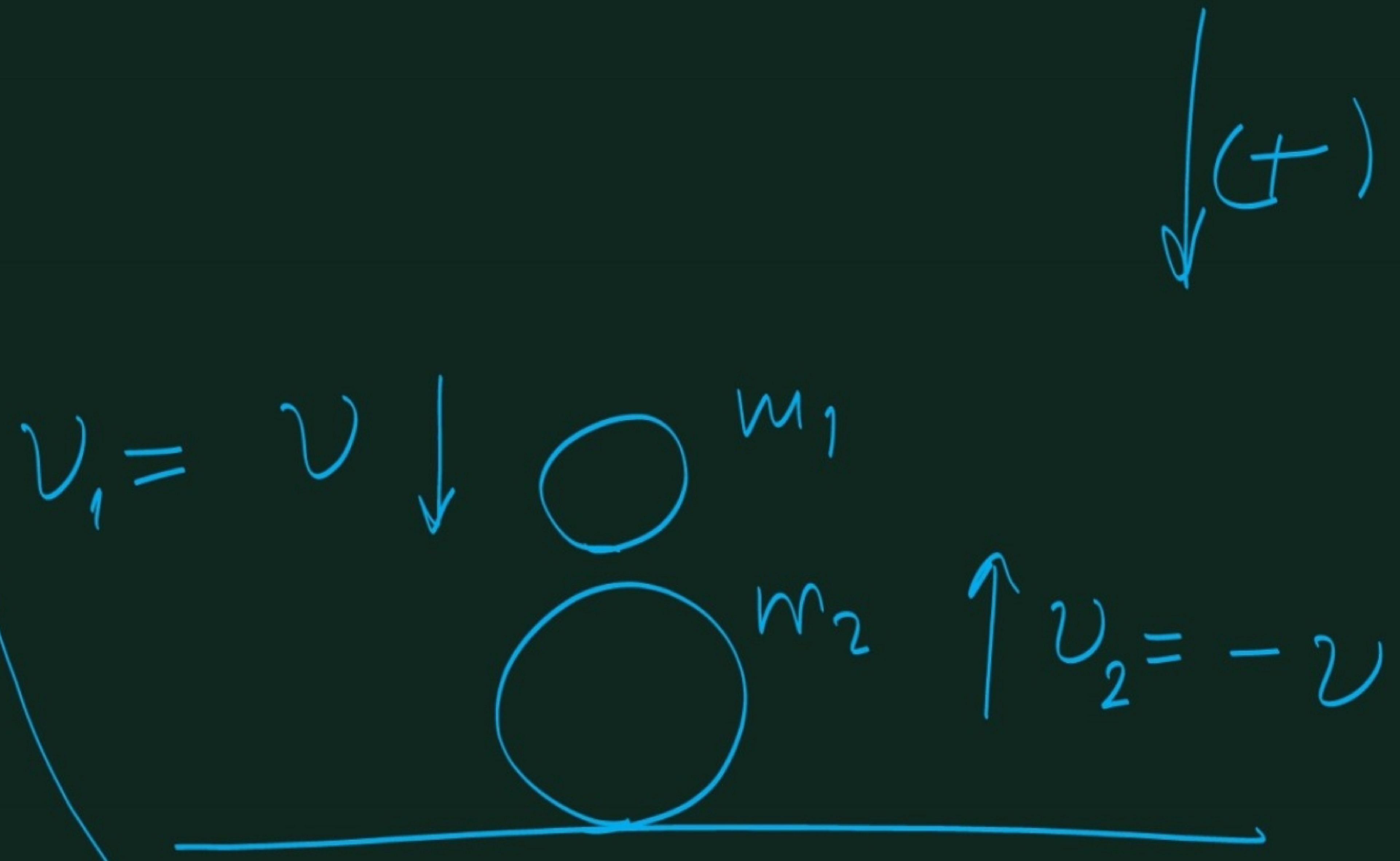
ΑΔΕ: $m_1 g h = \frac{1}{2} m v^2 \Rightarrow$

$v = \sqrt{2gh}$.



Οι μπάλες φτάνουν στο έδαφος με ταχύτητα v προς τα κάτω.

Η m_2 χτυπά ελαστικά στο έδαφος οπότε αρχίζει να κινείται προς τα πάνω με ταχύτητα $v_2 = -v$ ενώ η m_1 ακόμα κατεβαίνει με ταχύτητα v.



$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

$m_2 >> m_1$

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

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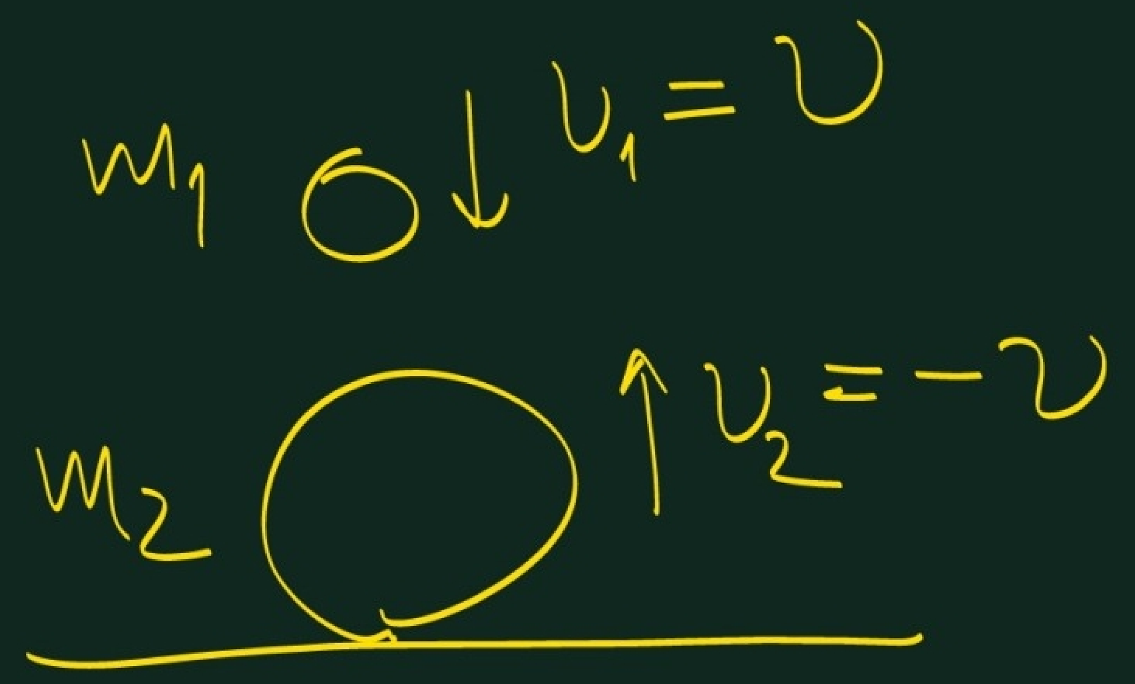
$$m_2 >> m_1$$



$$v_1' \simeq \frac{-m_2}{m_2} v_1 + \frac{2m_2}{m_2} v_2 \Rightarrow$$

$$\Rightarrow v_1' \simeq -v_1 + 2v_2$$

$$v_2' \simeq \frac{2m_1}{m_2} v_1 + \frac{m_2}{m_2} v_2 \Rightarrow$$

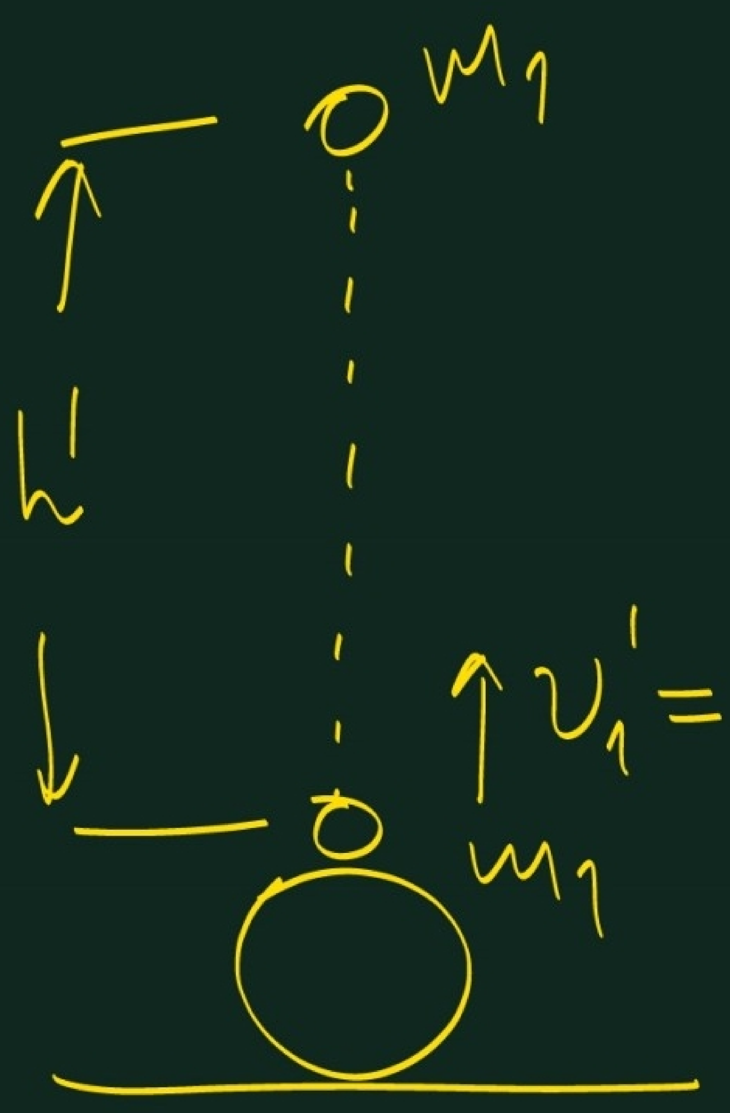
$$v_2' \simeq v_2$$

Άρα: $v_1' = -v + 2(-v) \Rightarrow v_1' = -3v$

και $v_2' = v_2 \Rightarrow v_2' = -v$

Άρα: $v_1' = -3v$

$v_2' = -v$



$$m_1 g h' = \frac{1}{2} m_1 v_1'^2 \Rightarrow$$

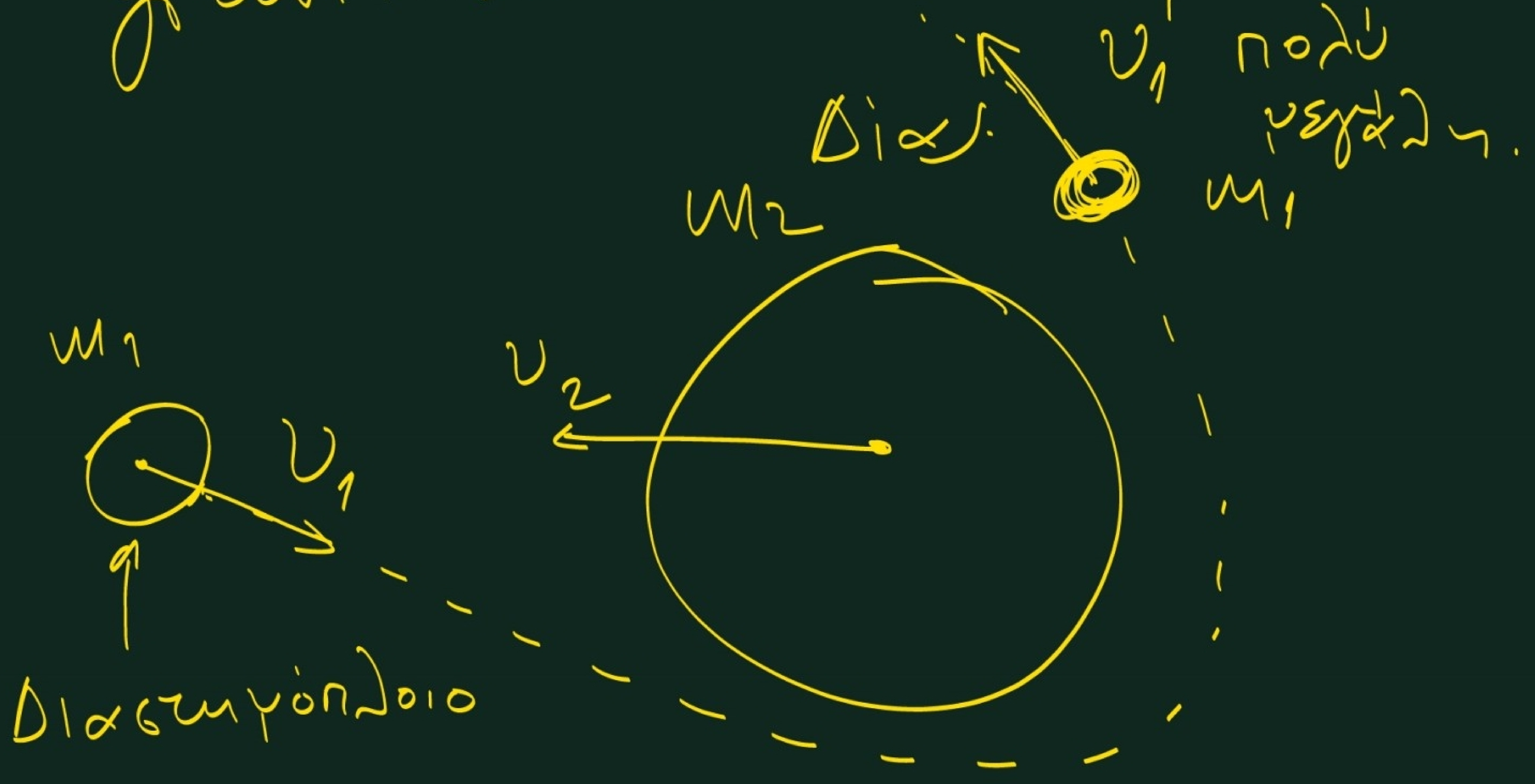
$$g h' = \frac{1}{2}(-3v)^2 \Rightarrow$$

$$g h' = \frac{1}{2}\cdot 9v^2 \Rightarrow$$

$$g h' = \frac{1}{2}\cdot 9 \cdot 2gh \Rightarrow$$

$$h' = 9h$$

Πίσω από αυτές τις μπάλες έχουμε την ιδέα της gravitational assist



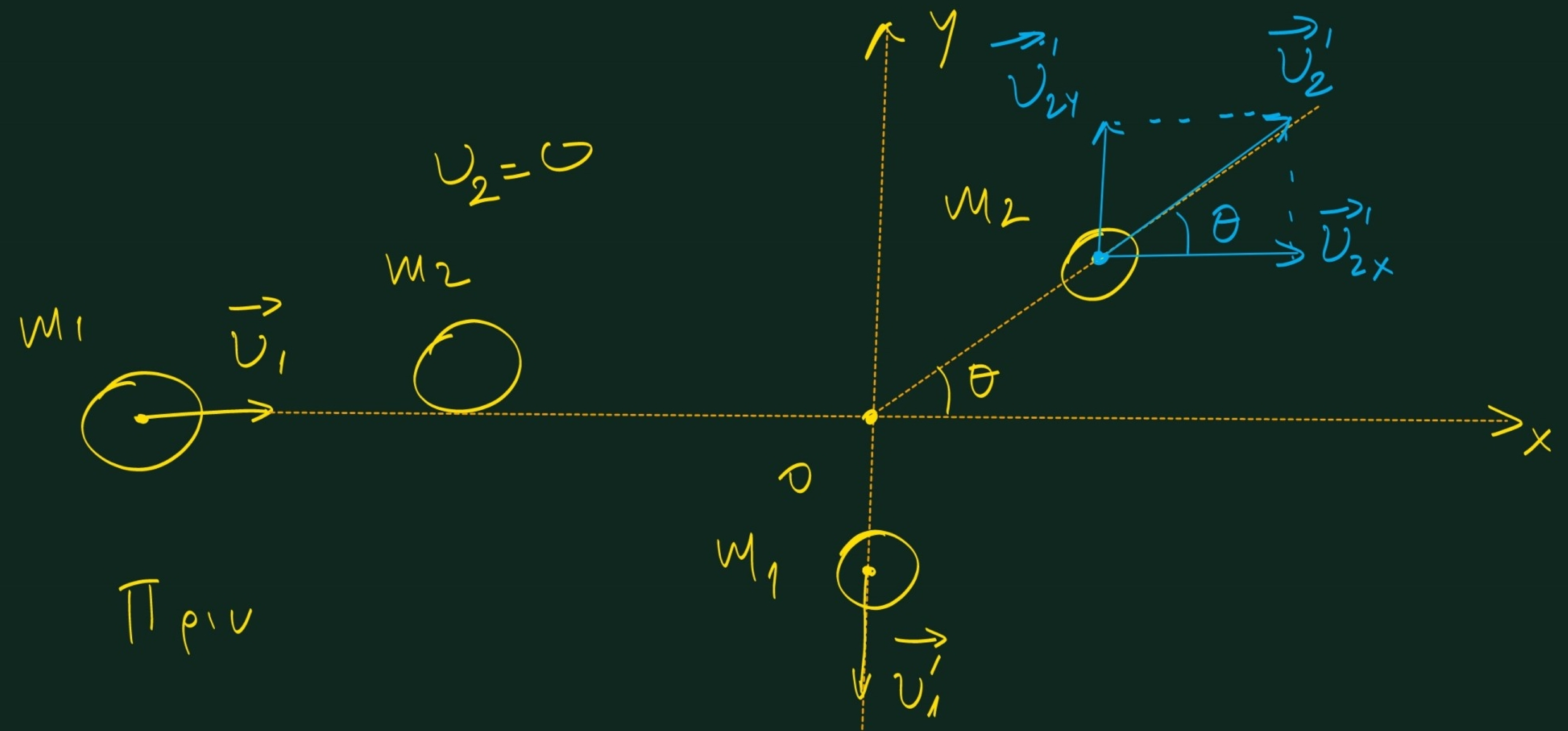
Ασκ. 3.54

$m_1 = 1\,kg$

$v_1 = 10\sqrt{3}\ m/s$

$m_2 = 2\,kg$

Μετά την κρούση η m_1 κινείται κάθετα προς την αρχική της διεύθυνση



$$P_{x,\pi\rho\iota\nu} = P_{x,\mu\varepsilon\tau\acute{\alpha}} \Rightarrow m_1 v_1 = m_2 v_{2x} \Rightarrow m_1 v_1 = m_2 v_2' \sigma\upsilon\nu\theta \Rightarrow v_1 = 2 v_2' \sigma\upsilon\nu\theta \quad (1)$$

$$P_{y,\pi\rho\iota\nu} = P_{y,\mu\varepsilon\tau\acute{\alpha}} \Rightarrow 0 = m_2 v_{2y}' - m_1 v_1' \Rightarrow m_2 v_{2y}' = m_1 v_1' \Rightarrow$$

$$2 v_2' \eta\mu\theta = v_1' \quad (2)$$

Η κρούση είναι ελαστική, άρα:

$$K_{\pi\rho\iota\nu} = K_{\mu\varepsilon\tau\acute{\alpha}} \Rightarrow$$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \Rightarrow \frac{1}{2} \cdot 1 \cdot 300 = \frac{1}{2} \cdot 1 \cdot v_1'^2 + \frac{1}{2} \cdot 2 \cdot v_2'^2 \Rightarrow$$

$$2 v_2'^2 + v_1'^2 = 300 \quad (3)$$

$$v_1 = 2v_2' \,\sigma\upsilon\nu\theta \quad (1)$$

$$v_1' = 2v_2' \,\eta\mu\theta \quad (2)$$

$$2v_2'^2 + v_1'^2 = 300 \quad (3)$$

$$(v_1 = 10\sqrt{3}\ m/s)$$

Άρα: $v_1'^2 + 2v_2'^2 = 300 \Rightarrow$

$v_1'^2 + 200 = 300 \Rightarrow$

$v_1'^2 = 100 \Rightarrow v_1' = 10\,m/s.$

Υψώνουμε τις (1) κ (2) στο τετράγωνο και προσθέτουμε:

$$4v_2'^2 \sigma\upsilon\nu^2\theta + 4v_2'^2 \eta\mu^2\theta = v_1^2 + v_1'^2 \Rightarrow$$

$$4v_2'^2 = 300 + v_1'^2 \Rightarrow \begin{cases} -v_1'^2 + 4v_2'^2 = 300 \\ v_1'^2 + 2v_2'^2 = 300 \end{cases} \; (+)$$

$$6v_2'^2 = 600 \Rightarrow v_2'^2 = 100 \Rightarrow v_2' = 10\,m/s.$$

$v_1 = 2v_2' \,\sigma\upsilon\nu\theta \Rightarrow$

$10\sqrt{3} = 2 \cdot 10 \cdot \sigma\upsilon\nu\theta \Rightarrow$

$\sigma\upsilon\nu\theta = \dfrac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ$

β) $K_{1,\pi\rho\iota\nu} = \frac{1}{2} m_1 \upsilon_1^2 = \frac{1}{2} m_1 \upsilon_0^2 =$

$= \frac{1}{2} \cdot 1 \cdot 300 = 150\ J.$

$K_{1,\mu\varepsilon\tau\acute{\alpha}} = \frac{1}{2} m_1 \upsilon_1'^2 = \frac{1}{2} \cdot 1 \cdot 100 \Rightarrow$

$K_{1,\mu\varepsilon\tau\acute{\alpha}} = 50\ J.$

$$\Pi\% = \frac{\Delta K_1}{K_{1,\pi\rho\iota\nu}} \cdot 100 = \frac{K_{1,\mu\varepsilon\tau\acute{\alpha}} - K_{1,\pi\rho\iota\nu}}{K_{1,\pi\rho\iota\nu}} \cdot 100 =$$

$$= \frac{50-150}{150} \cdot 100 = \frac{-100}{150} \cdot 100 = -66{,}7\%$$

Άρα η απώλεια είναι 66,7%

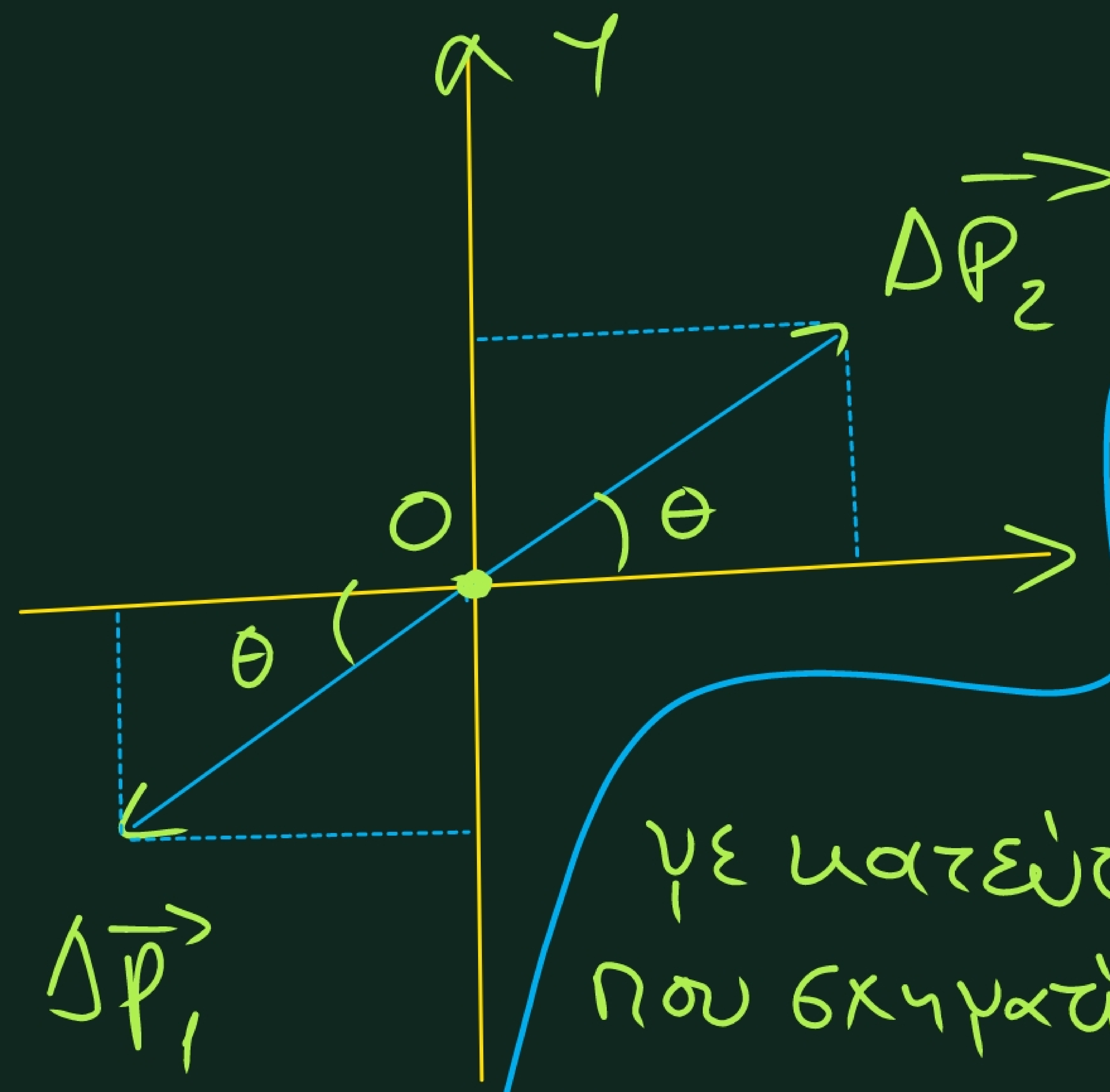
γ) $\vec{p}_1 + \vec{p}_2 = \sigma\tau\alpha\theta \Rightarrow$

$\Delta\vec{p}_1 + \Delta\vec{p}_2 = \vec{0} \Rightarrow$

$\Delta\vec{p}_2 = -\Delta\vec{p}_1$

$\Delta\vec{p}_2 = \vec{p}_{2,\mu\varepsilon\tau\acute{\alpha}} - \vec{p}_{2,\pi\rho\iota\nu} = m_2\vec{\upsilon}_2' - \vec{0} = m_2\vec{\upsilon}_2'.$

$|\Delta\vec{p}_2| = m_2|\vec{\upsilon}_2'| = m_2\upsilon_2' = 2 \cdot 10 \Rightarrow |\Delta\vec{p}_2| = 20\ kg\cdot m/s.$



Άρα $|\Delta\vec{p}_1| = |\Delta\vec{p}_2| =$ 20 kg·m/s με κατεύθυνση που σχηματίζει γωνία $\theta = 30^\circ$ με τον άξονα x όπως στο σχήμα.

δ) $\Delta\vec{p}_1 + \Delta\vec{p}_2 = \vec{0} \Rightarrow$

$$\frac{\Delta\vec{p}_1}{\Delta t} + \frac{\Delta\vec{p}_2}{\Delta t} = \vec{0} \Rightarrow$$

$$\vec{F}_1 + \vec{F}_2 = 0 \Rightarrow$$

$$\vec{F}_2 = -\vec{F}_1.$$

$$|\vec{F}_1| = |\vec{F}_2| = \frac{|\Delta\vec{p}_2|}{\Delta t} = \frac{20}{0{,}1} = 200N$$

Άρα οι δυνάμεις $\vec{F}_1$ και $\vec{F}_2$ έχουν μέτρο 200N και σχηματίζουν γωνία 30° με τον άξονα x, όπως στο σχήμα: