## Ασκ. 5.219 (Μαθ)

$\ell = OA = 1{,}6\,m$

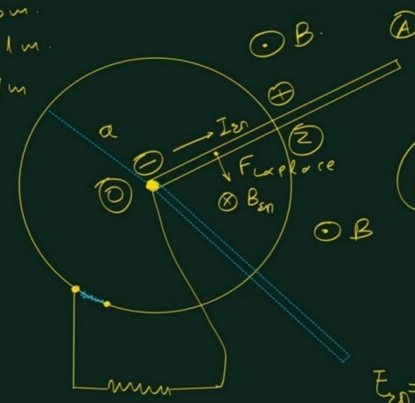
$OZ = a = 1\,m$

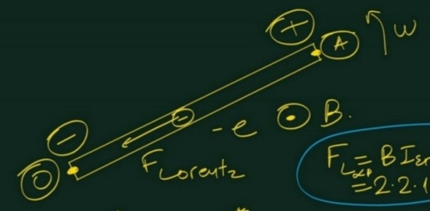
$R^* = 0{,}4\,\Omega/m$ για τον OA.

$V_1 = 0{,}2\,V$

$R_1 = 0{,}1\,\Omega$

$B = 2\,T$

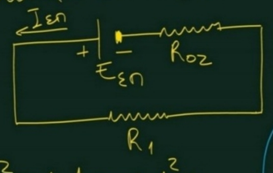



$F_{L_{\varepsilon\pi}} = B I_{\varepsilon\pi} a = 2 \cdot 2 \cdot 1 \Rightarrow F_{L_{\varepsilon\pi}} = 4\,N$

α) $R_{OZ} = R^* \cdot OZ = R^* \cdot a = 0{,}4 \cdot 1 = 0{,}4\,\Omega$

β) Ισοδύναμο κύκλωμα.




$V_1 = I_{\varepsilon\pi} R_1 \Rightarrow$

$0{,}2 = I_{\varepsilon\pi} \cdot 0{,}1 \Rightarrow$

$I_{\varepsilon\pi} = 2\,A$

$E_{\varepsilon\pi} = I_{\varepsilon\pi}(R_1 + R_{OZ}) \Rightarrow$

$E_{\varepsilon\pi} = 2 \cdot (0{,}1 + 0{,}4) \Rightarrow$

$E_{\varepsilon\pi} = 1\,Volt$

$E_{\varepsilon\pi} = \frac{1}{2} B \omega a^2 \Rightarrow 1 = \frac{1}{2} \cdot 2 \cdot \omega \cdot 1^2 \Rightarrow$

$\omega = 1\,rad/sec$

γ) $\omega = 1\,rad/sec \Rightarrow$

$\frac{2\pi}{T} = 1 \Rightarrow T = 2\pi\ sec$

$Q = I_{\varepsilon\varphi}^2 (R_1 + R_{o2}) \cdot t \Rightarrow$

$Q = I_{\varepsilon\varphi}^2 (R_1 + R_{o2}) \cdot 10T \Rightarrow$

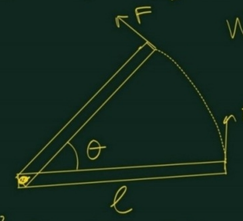
$Q = 2^2 \cdot (0{,}1 + 0{,}4) \cdot 10 \cdot 2\pi \Rightarrow$

$Q = 4 \cdot 0{,}5 \cdot 10 \cdot 2\pi \Rightarrow$

$Q = 40\pi$ Joule

δ) Το έργο της σταθερής δύναμης F που ασκείται κάθετα στο άκρο Α του δοκού γίνεται θερμότητα στις αντιστάσεις.

Δηλαδή: $P_F = P_{R_{o1}}$ (εξίσωση ισχύων)




$W_F = F \cdot \ell \cdot \theta \Rightarrow$

$\frac{\Delta W_F}{\Delta t} = F \cdot \ell \cdot \frac{\Delta\theta}{\Delta t} = F \cdot \ell \cdot \omega$

$P_{R_{o1}} = I_{\varepsilon\varphi}^2 (R_1 + R_{o2})$

Άρα: $I_{\varepsilon\varphi}^2 (R_1 + R_{o2}) = F \cdot \ell \cdot \omega \Rightarrow 2^2 \cdot (0{,}1 + 0{,}4) = F \cdot 1{,}6 \cdot 1 \Rightarrow$

$2 = 1{,}6F \Rightarrow F = \frac{2}{1{,}6} \Rightarrow F = 1{,}25\,N$

## Ασκ. 5.220 (Εξετάσεις 1993)

$L_1 = 1\,m$

$L_2 = 2\,m$

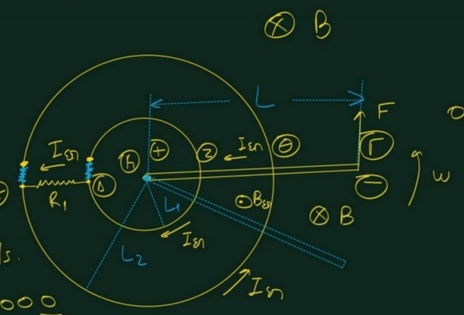
$B = 5\,T$

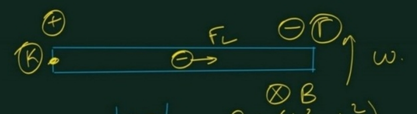
$R_1 = 600\,\Omega$

$L = 2{,}5\,m$

$\omega = 10\,rad/s$

$R_{K\Gamma} = 1000\,\Omega$



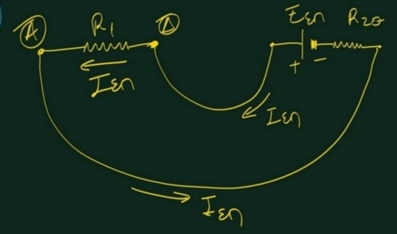

α) $E_{\varepsilon\pi} = \dfrac{|\Delta\varphi|}{\Delta t} = \dfrac{B\pi(L_2^2 - L_1^2)}{T} =$

$= \dfrac{B\pi(L_2^2 - L_1^2)}{\frac{2\pi}{\omega}} = \dfrac{1}{2} B\omega(L_2^2 - L_1^2)$

γιατί $V_Z > V_\Theta$

Άρα: $E_{\varepsilon\pi} = \frac{1}{2} \cdot 5 \cdot 10 \cdot (4-1) = 25 \cdot 3 \Rightarrow E_{\varepsilon\pi} = 75\,Volt$

β)




Άρα το $I_{ση}$ έχει φορά από το Δ στο Α και είναι:

$$I_{ση} = \frac{E_{ση}}{R_1 + R_{zθ}} \Rightarrow$$

$$I_{ση} = \frac{75}{1000} \Rightarrow$$

$$I_{ση} = 0{,}075\,A.$$

$$R_{zθ} = \frac{zθ}{κr} \cdot R_{κr} \Rightarrow$$

$$R_{zθ} = \frac{L_2 - L_1}{L} \cdot R_{κr} \Rightarrow$$

$$R_{zθ} = \frac{2-1}{2{,}5} \cdot 1000 \Rightarrow$$

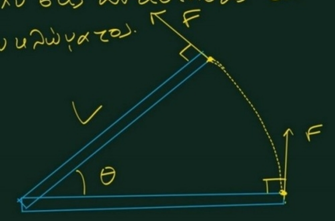
$$R_{zθ} = 400\,\Omega$$

γ) $V_Δ - I_{ση} \cdot R_1 = V_A \Rightarrow$

$V_A - V_Δ = -I_{ση} \cdot R_1 \Rightarrow$

$V_{AΔ} = -\frac{75}{1000} \cdot 600 \Rightarrow V_{AΔ} = -45\,Volt$

δ) Η ισχύς της δύναμης F είναι ίση με την ηλεκτρική ισχύ της αντίστασης του κυκλώματος.




$$W_F = F \cdot L \cdot \theta$$

$$\frac{\Delta W_F}{\Delta t} = F \cdot L \cdot \frac{\Delta \theta}{\Delta t} \rightarrow$$

$$P_F = F \cdot L \cdot \omega$$

$$P_F = P_{R_{ολ}} \Rightarrow$$

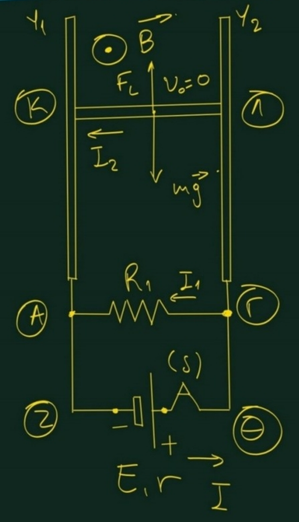
$$F \cdot L \cdot \omega = I_{ef}^2 (R_{20} + R_1) \Rightarrow$$

$$F \cdot 2{,}5 \cdot 10 = \left(\frac{75}{1000}\right)^2 \cdot 1000 \Rightarrow$$

$$1 \quad \cancel{25}F = \frac{\overset{3}{\cancel{75}} \cdot 75}{1000} \Rightarrow$$

$$F = \frac{225}{1000} \Rightarrow F = 0{,}225\,N$$

## Ασκ. 5.219 (Μαθ)




$m = 0{,}5\,kg$

$\ell = 1\,m$

$R_{K\Lambda} = R$

Ο αγωγός ΚΛ αρχικά ισορροπεί ($\upsilon_0 = 0$)

$\vec{g}$

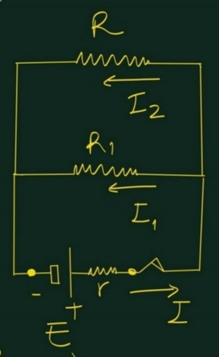
$E = 8\,Volt$

$r = 0{,}45\,\Omega$

$R_1 = 3\,\Omega$

$B = 1\,T$

α) Το $\vec{B}$ πρέπει να έχει φορά προς τα έξω επειδή η $F_L$ στον αγωγό έχει φορά προς τα πάνω.




$F_L = mg \Rightarrow$

$B I_2 \ell = mg \Rightarrow$

$1 \cdot I_2 \cdot 1 = 5 \Rightarrow$

$I_2 = 5\,A$

$I_2 R = I_1 R_1 \Rightarrow$

$5R = I_1 \cdot 3 \Rightarrow$

$I_1 = \frac{5R}{3}$

$$I = I_1 + I_2 = \left(\frac{5R}{3} + 5\right)$$

$$I = \frac{E}{R_{\sigma\upsilon}} = \frac{E}{r + \frac{R_1 R}{R_1 + R}} = \frac{E(R_1 + R)}{r(R_1 + R) + R_1 R}$$

$$I = \frac{5R}{3} + 5$$

$$I = \frac{E(R_1+R)}{r(R_1+R)+R_1R} = \frac{8(3+R)}{0{,}45(3+R)+3R}$$

$$\frac{5R+15}{3} = \frac{8(R+3)}{0{,}45(3+R)+3R} \Rightarrow$$

$$\frac{5\cancel{(R+3)}}{3} = \frac{8\cancel{(R+3)}}{0{,}45(R+3)+3R} \Rightarrow$$

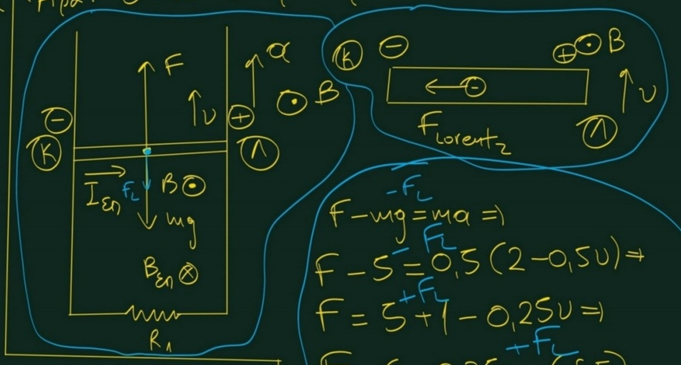
$$2{,}25\cdot(R+3)+15R = 24 \Rightarrow$$

$$17{,}25R = 24-6{,}75 \Rightarrow 17{,}25R = 17{,}25 \Rightarrow R = 1\,\Omega$$

β) i) $\alpha = 2-0{,}5\upsilon$ (SI)

Η ταχύτητα γίνεται οριακή και σταθερή όταν γίνει $\alpha = 0$

Άρα: $0 = 2-0{,}5\upsilon_{op} \Rightarrow \upsilon_{op} = 4\,m/s$ (προς τα πάνω)







$$F - mg \overset{-F_L}{=} ma \Rightarrow$$

$$F - 5 \overset{-F_L}{=} 0{,}5(2-0{,}5\upsilon) \Rightarrow$$

$$F = 5 \overset{+F_L}{+} 1 - 0{,}25\upsilon \Rightarrow$$

$$F = 6 - 0{,}25\upsilon \overset{+F_L}{} \text{ (SI)}$$

$F_L = 0{,}25\upsilon$

$F = 6N$

$P_F = \vec{F}\cdot\vec{\upsilon} = F\cdot\upsilon$

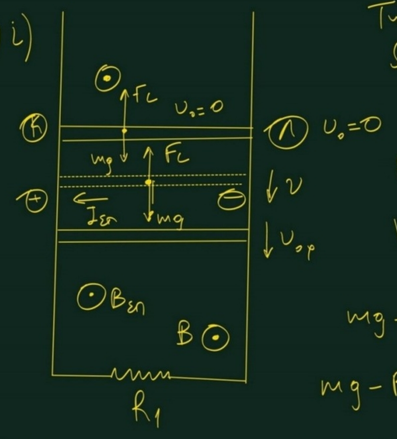
αφού $\vec{F} \uparrow\uparrow \vec{\upsilon}$.

Άρα: $P_F = (6 - 0{,}25\upsilon)\cdot\upsilon$

Άρα για $\upsilon = 1\,m/s$ έχουμε:

$P_F = 6 - 0{,}25 \Rightarrow P_F = 5{,}75\,Watt$

γ) i)




Τη στιγμή $t=0$ ο αγωγός διαρρέεται από το ρεύμα $I_2$

$F_L = B I_2 \ell = 5N$

$W = mg = 5N$

$mg - F_L = ma \Rightarrow$

$mg - B I_{\varepsilon\pi} \ell = ma \Rightarrow$

$mg - B\cdot\frac{E_{\varepsilon\pi}}{R_1 + R}\ell = ma \Rightarrow$

$mg - \frac{B^2\ell^2\upsilon}{R_1+R} = ma \Rightarrow$

$5 - \frac{\upsilon}{4} = 0{,}5a \Rightarrow 0{,}5a = 5 - 0{,}25\upsilon \Rightarrow$

$a = 10 - 0{,}5\upsilon$ (SI)

$a = 0 \Rightarrow \upsilon = \upsilon_{op} = 20\,m/s$

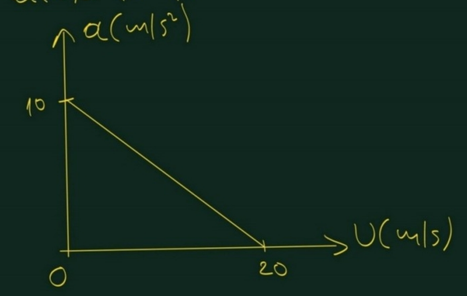
προς τα κάτω

$a = 10 - 0{,}5\,v$

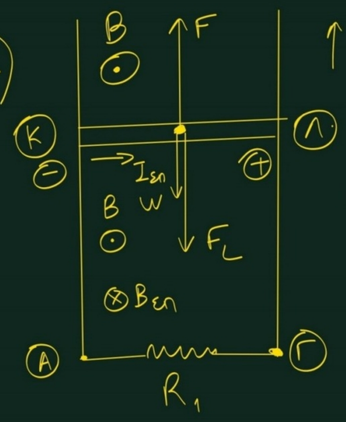
$a = 0 \Rightarrow v = v_{op} = 20\,m/s$.

Αφού $v \in [0, 20]$

$a(0) = 10\,m/s^2$




β) ii)




$\uparrow a \uparrow v$.

$\uparrow (+)$

$F - F_L - mg = ma$

$F_L = BI_{επ}l = B\dfrac{Bvl}{R_1 + R}l =$

$= 1 \cdot \dfrac{1 \cdot v \cdot 1}{4} \cdot 1 = \dfrac{v}{4} \Rightarrow$

$F_L = 0{,}25\,v\,(SI)$

Άρα: $F - F_L - mg = ma \Rightarrow F - 0{,}25v - 0{,}5 \cdot 10 = 0{,}5(2 - 0{,}5v)$

$\Rightarrow F - \cancel{0{,}25v} - 5 = 1 - \cancel{0{,}25v} \Rightarrow F = 6N$ (σταθερή)

$P_F = F \cdot v_1 = 6 \cdot 1 \Rightarrow P_F = 6\,Watt$

## Αυτεπαγωγή πηνίου.

$$E_{αυτ} = -L\frac{\Delta i}{\Delta t}$$

Για σωληνοειδές:

$$B = \mu_0 n i$$

$$E_{επ} = -N\frac{\Delta \Phi}{\Delta t} = -N\cdot\mu_0 n\cdot\frac{\Delta i}{\Delta t}\cdot S$$

$$E_{επ} = E_{αυτ} \Rightarrow -L\cdot\frac{\Delta i}{\Delta t} = -N\mu_0 n\frac{\Delta i\cdot S}{\Delta t} \Rightarrow$$

$$L = N\mu_0 n = \mu_0\frac{N\cdot S}{\ell}\cdot\ell \Rightarrow L = \mu_0\frac{N^2}{\ell}\cdot S$$

μεγό

$$-L\cdot\frac{\Delta i}{\cancel{\Delta t}} = iV = \frac{\Delta q}{\cancel{\Delta t}}V$$

Αν στο εσωτερικό του πηνίου υπάρχει υλικό μαγνητικής διαπερατότητας μ θα έχουμε:

$$B = \mu\mu_0 n i \text{ και}$$

$$L = \mu\mu_0\frac{N^2}{\ell}\cdot S.$$

Ασκ. 5.66 σελ. 229.

$l = 0{,}5\ m$

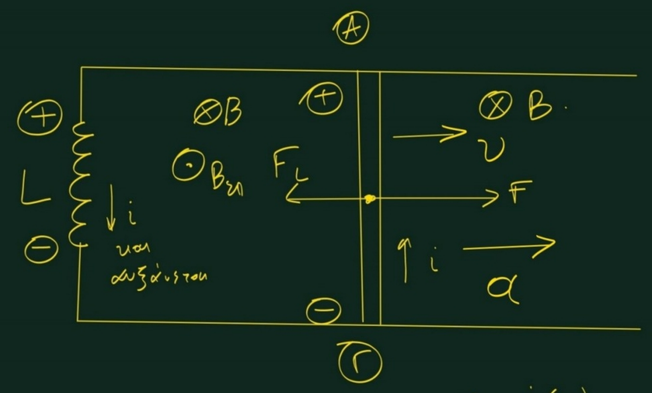
$L = 25 \cdot 10^{-3}$ Henry (Πηνίο)

$m = 150g = 0{,}15\ kg$

$R = 5 \cdot 10^{-2}\ \Omega$

$B = 0{,}1T$

Αγνοούμε δύναμη F τέτοια ώστε: $i = 2t + 3 (SI)$




α) $E_{αυτ} = -L \frac{\Delta i}{\Delta t} = -L \cdot \frac{i(t) - i(0)}{t - 0} =$

$= -L \cdot \frac{2t + 3 - (2 \cdot 0 + 3)}{t} = -2L = -2 \cdot 25 \cdot 10^{-3} \Rightarrow$

$E_{αυτ} = -50 \cdot 10^{-3} \Rightarrow |E_{αυτ}| = 0{,}05\ Volt$

b) $E_{en} - iR = |E_{aut}| \Rightarrow$

$$E_{en} - iR = L \cdot \frac{\Delta i}{\Delta t} \Rightarrow$$

$$Bvl - iR = L \cdot \frac{\Delta i}{\Delta t} \Rightarrow$$

$$0{,}1 \cdot v \cdot 0{,}5 - i \cdot 5 \cdot 10^{-2} = 25 \cdot 10^{-3} \cdot \frac{\Delta i}{\Delta t} \Rightarrow$$

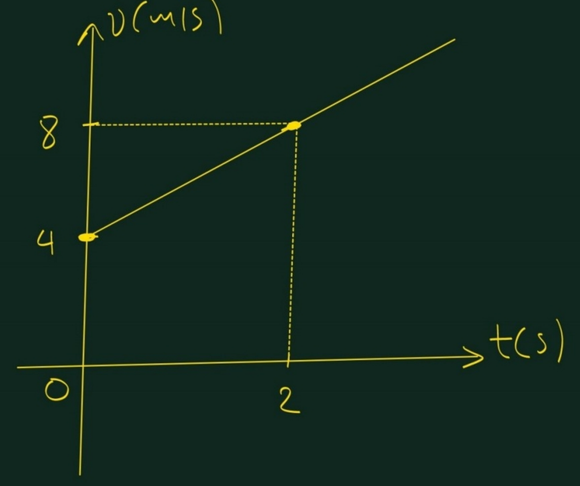
$$5 \cdot \cancel{10^{-2}} \cdot v - 5 \cdot \cancel{10^{-2}} \cdot i = 2{,}5 \cdot \cancel{10^{-2}} \cdot \frac{\Delta i}{\Delta t} \Rightarrow$$

$$2v - 2i = \frac{\Delta i}{\Delta t} \Rightarrow 2v - 2 \cdot (2t+3) = 2 \Rightarrow$$

$$v - 2t - 3 = 1 \Rightarrow v = 4 + 2t \ (SI)$$

$v(0) = 4\ m/s.$

$v(2) = 8\ m/s.$



γ) $v(t) = 2t + 4$

$$\frac{\Delta v}{\Delta t} = \frac{v(t) - v(0)}{t - 0} =$$

$$= \frac{2t + 4 - (2 \cdot 0 + 4)}{t} =$$

$$= \frac{2t}{t} = 2\,m/s^2 \Rightarrow$$

$$\alpha = 2\,m/s^2 \text{ (σταθερή)}$$

δ) $\Sigma F = ma \Rightarrow F - F_L = ma \Rightarrow$

$$F - Bil = ma \Rightarrow$$

$$F - 0{,}1(2t+3) \cdot 0{,}5 = 0{,}15 \cdot 2 \Rightarrow$$

$$F - 0{,}1t - 0{,}15 = 0{,}3 \Rightarrow$$

$$F = 0{,}1t + 0{,}45 \quad (SI)$$

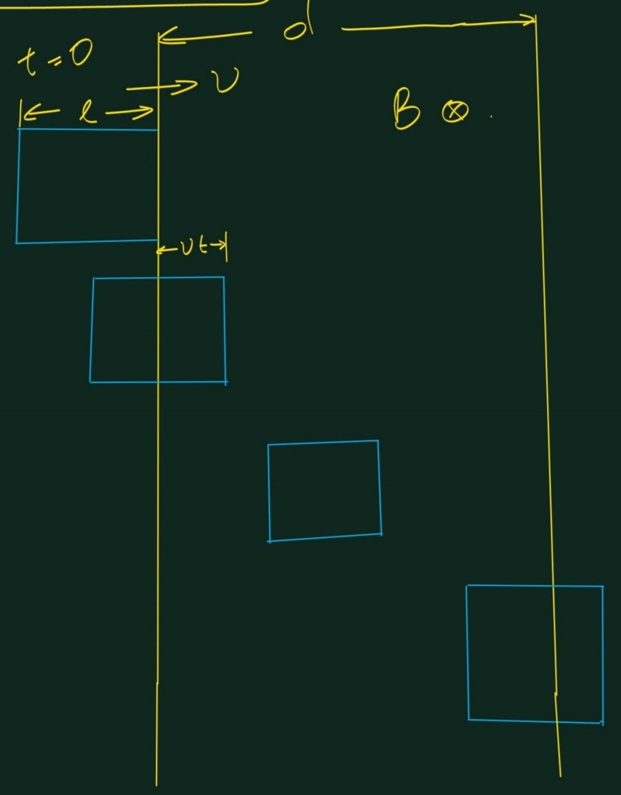
$$P_F = \vec{F} \cdot \vec{v} = F \cdot v = (0{,}1t + 0{,}45) \cdot (2t + 4)$$

Άρα: $P_F(4) = (0{,}1 \cdot 4 + 0{,}45) \cdot (8 + 4) =$

$$= 0{,}85 \cdot 12 = 0{,}8 \cdot 12 + 0{,}05 \cdot 12 =$$

$$= 9{,}6 + 0{,}6 = 10{,}2 \text{ Watt}$$

Ασκ. 5.67 σελ. 226.




α) $t=0$ εισαγωγή.

$t=t_1=\frac{\ell}{\upsilon}$ (χρόνος εισόδου)

$t_2=\frac{d-\ell}{\upsilon}$ (χρόνος παραμονής)

$t_3=\frac{\ell}{\upsilon}$ (χρόνος εξόδου)

$\Phi_{max}=B\ell^2=0,4\cdot 0,25^2=0,4\cdot 0,25\cdot 0,25=$
$=0,1\cdot 0,25=0,025=$
$=2,5\cdot 10^{-2}\ Wb.$

$t_1=\frac{0,25}{0,05}=5sec$

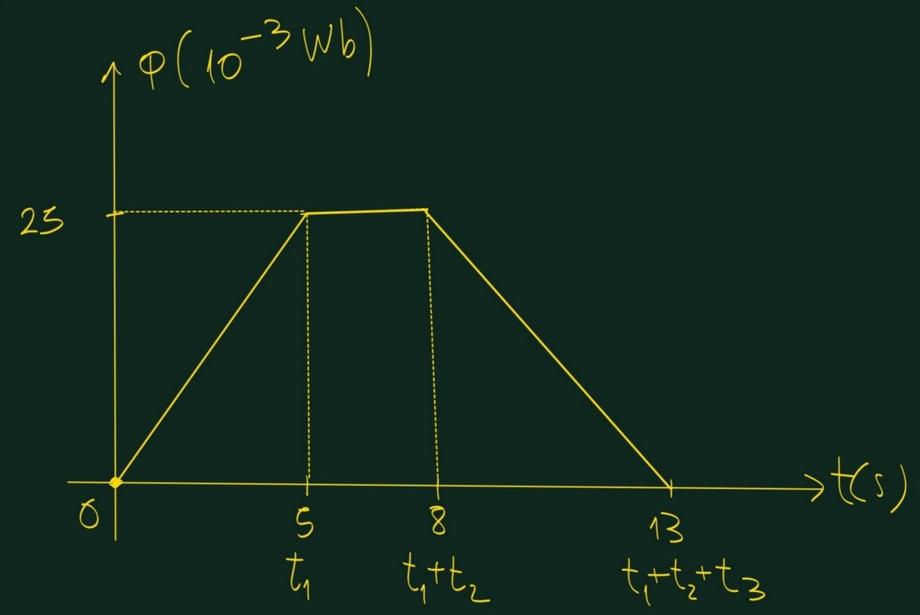
$t_2=\frac{0,4-0,25}{0,05}=3sec$

$t_3=5sec.$

$$\Phi = B\,l\cdot v t = 0{,}4\cdot 0{,}25\cdot 0{,}05\cdot t = 0{,}1\cdot 0{,}05 t \Rightarrow$$

$$\Phi(t) = 5\cdot 10^{-3} t \quad (\text{Wb})$$

$$\Phi_{max} = 25\cdot 10^{-3}\ \text{Wb}.$$




$$\Phi = \lambda t + b$$

$$\Phi(8) = 25 \Rightarrow 8\lambda + b = 25$$

$$\Phi(13) = 0 \Rightarrow 13\lambda + b = 0 \Rightarrow$$

$$b = -13\lambda.$$

$$8\lambda - 13\lambda = 25 \Rightarrow \lambda = -5$$

$$b = +65.$$

$$\Phi(t) = -5t + 65 \quad (\text{mWb})$$

$$\Phi(t) = \begin{cases} 5t, & 0 \le t \le 5\,\text{sec} \\ 25, & 5 < t \le 8\,\text{sec} \\ -5t+65, & 8 < t \le 13\,\text{sec} \end{cases} \quad (\text{mWb})$$

B) $E_{\varepsilon n} = -N \cdot \frac{\Delta \Phi}{\Delta t}$

$N = 1$

Από $0 \to 5$ sec.

$$E_{\varepsilon n} = -1 \cdot \frac{\Phi(5) - \Phi(0)}{5 - 0} \Rightarrow$$

$$E_{\varepsilon n} = -\frac{25 \cdot 10^{-3} - 0}{5} \Rightarrow$$

$$E_{\varepsilon n} = -5 \cdot 10^{-3} \text{ Volt}$$

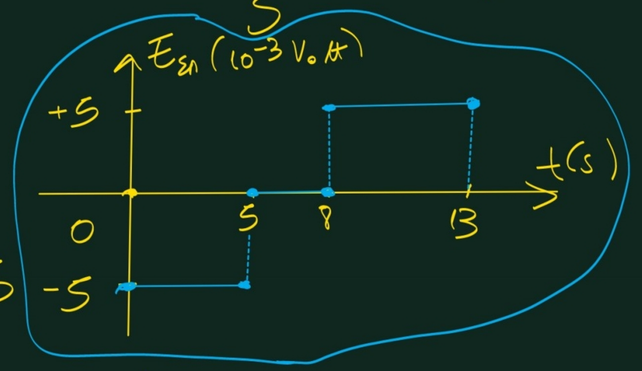
Από $5 \to 8$ sec:

$$E_{\varepsilon n} = -N \cdot \frac{\Delta \Phi}{\Delta t} = -1 \cdot \frac{\Phi(8) - \Phi(5)}{8 - 5} = -\frac{25 \cdot 10^{-3} - 25 \cdot 10^{-3}}{3} = 0 \text{ Volt}$$

Από $8 \to 13$ sec.

$$E_{\varepsilon n} = -N \cdot \frac{\Delta \Phi}{\Delta t} = -1 \cdot \frac{\Phi(13) - \Phi(8)}{13 - 8} = -\frac{0 - 25 \cdot 10^{-3}}{5} \Rightarrow$$

$$E_{\varepsilon n} = 5 \cdot 10^{-3} \text{ Volt}$$

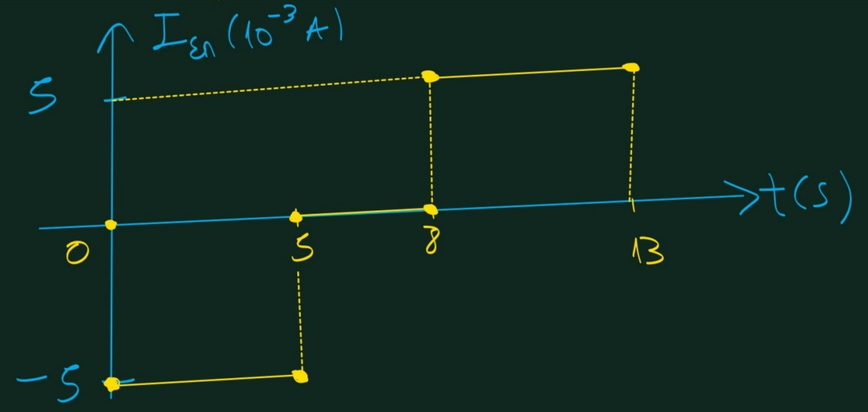



Άρα: $E_{\varepsilon n} = \begin{cases} -5 \cdot 10^{-3}, & 0 \le t \le 5 \\ 0, & 5 < t \le 8 \\ 5 \cdot 10^{-3}, & 8 < t \le 13 \end{cases}$ (SI)

γ) $R_{πλαισίου} = R^* \cdot 4l = 1 \cdot 4 \cdot 0{,}25 = 1\,\Omega$.

$I_{επ} = E_{επ} / R_{πλ}$.

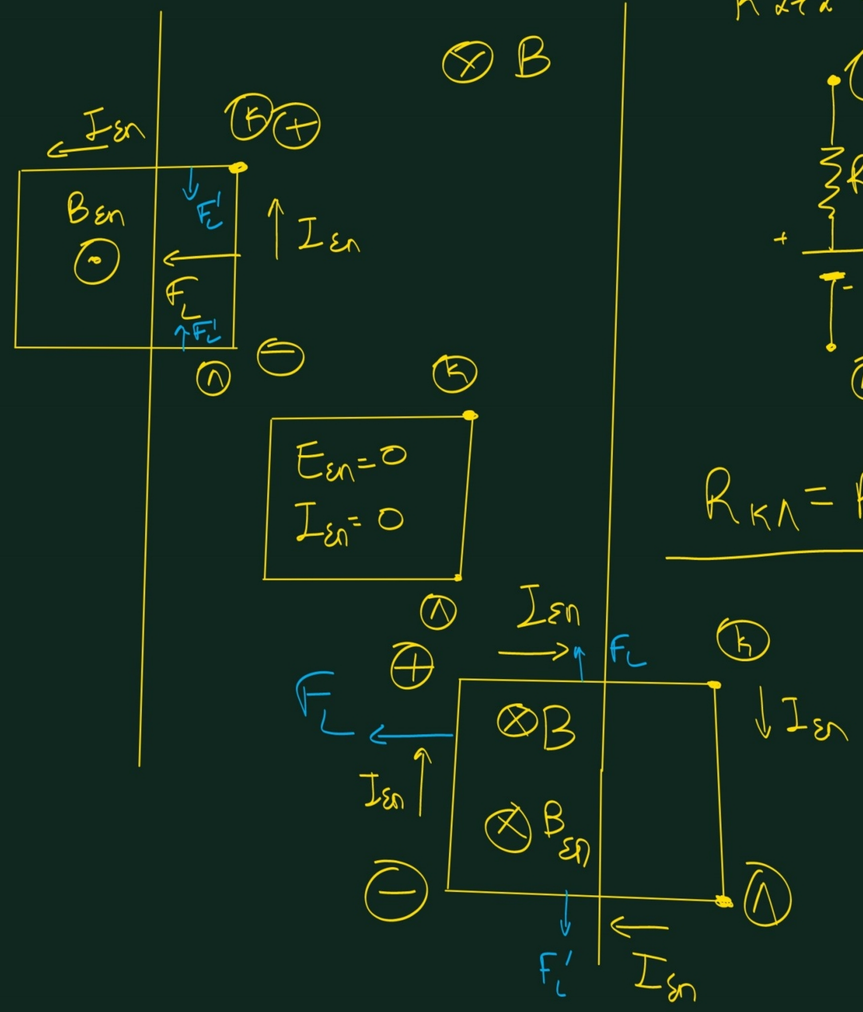
$$I_{επ} = \begin{cases} -5 \cdot 10^{-3}, & 0 \le t \le 5 \\ 0, & 5 < t \le 8 \\ 5 \cdot 10^{-3}, & 8 < t \le 13. \end{cases}$$



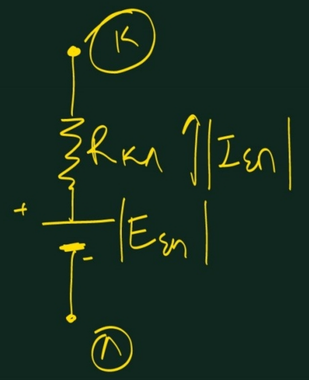

Το συνολικό φορτίο που διέρχεται από γι' αι διατομή του πλαισίου κατά την είσοδο και την έξοδο είναι μηδέν.

$$Q_{ολ} = \sum_{0}^{13} I_{επ} \cdot \Delta t = 0.$$

δ)




Κατά την είσοδο:




$$V_\Lambda + |E_{\varepsilon\eta}| - |I_{\varepsilon\eta}|\,R_{K\Lambda} = V_K \Rightarrow$$

$$V_K - V_\Lambda = |E_{\varepsilon\eta}| - |I_{\varepsilon\eta}|\cdot R_{K\Lambda} \Rightarrow$$

$$V_{K\Lambda} = 5\cdot 10^{-3} - 5\cdot 10^{-3}\cdot 0{,}25 \Rightarrow$$

$$R_{K\Lambda} = R^{*}\cdot \ell = 0{,}25\,\Omega \qquad V_{K\Lambda} = 3{,}75\cdot 10^{-3}\ \text{Volt}.$$

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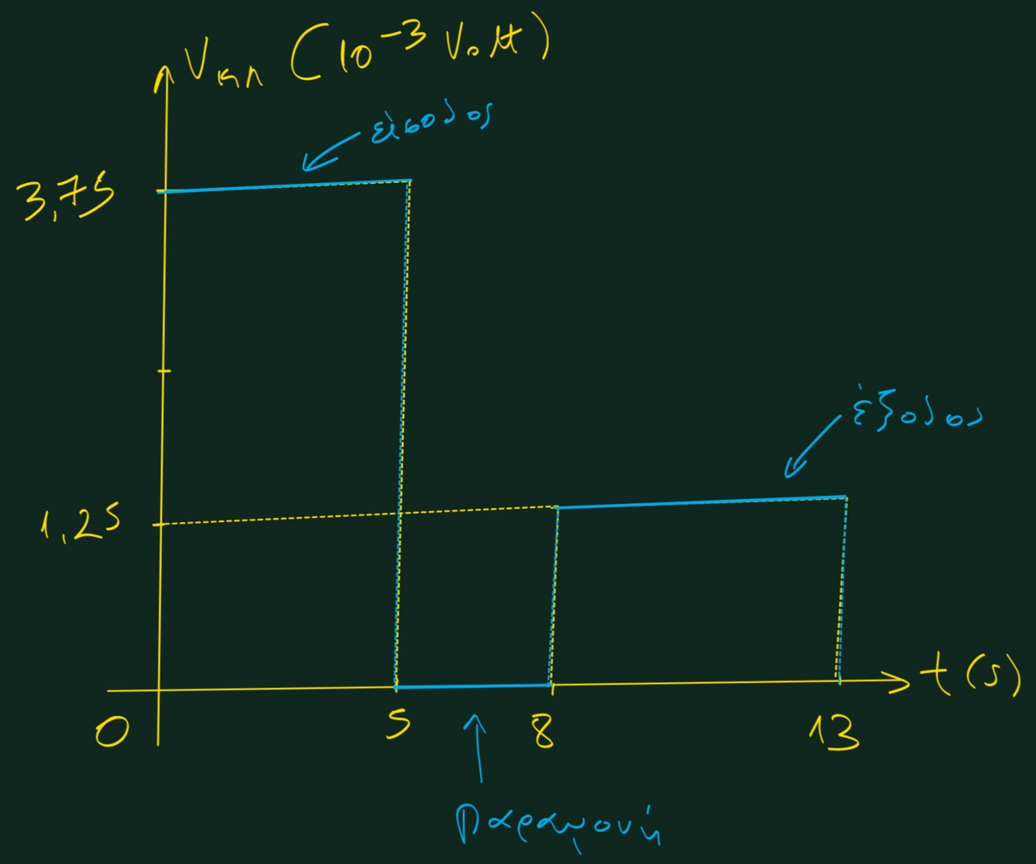
Κατά την έξοδο:

$$V_K - I_{\varepsilon\eta}\cdot R_{K\Lambda} = V_\Lambda \Rightarrow$$

$$V_K - V_\Lambda = I_{\varepsilon\eta}\cdot R_{K\Lambda} \Rightarrow$$

$$V_{K\Lambda} = 5\cdot 10^{-3}\cdot 0{,}25 \Rightarrow$$

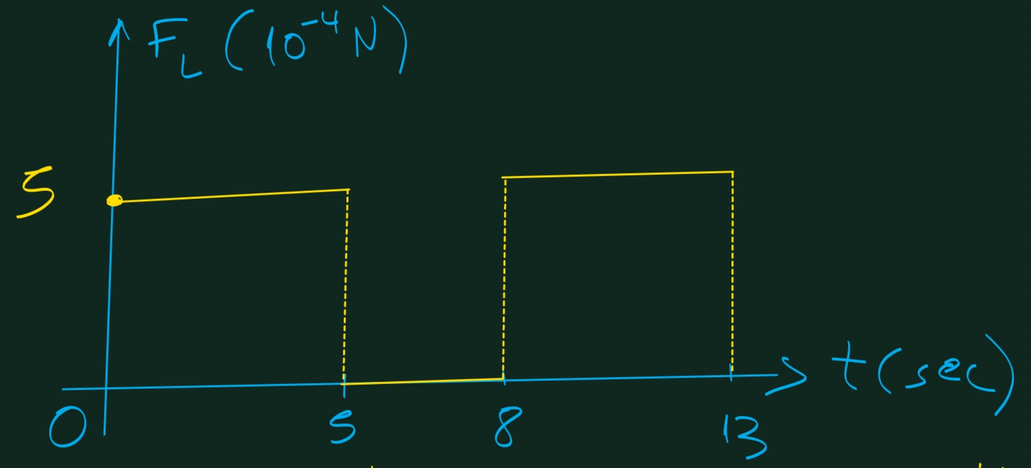
$$V_{K\Lambda} = 1{,}25\cdot 10^{-3}\ \text{Volt}$$




ε) Η $F_L$ έχει πάντα φορά προς τα αριστερά.

$$F_L = B I_{\epsilon\pi} \cdot \ell = 0{,}4 \cdot 5 \cdot 10^{-3} \cdot 0{,}25 =$$

$$= 0{,}1 \cdot 5 \cdot 10^{-3} \Rightarrow F_L = 5 \cdot 10^{-4}\ N.$$




Κατά την είσοδο: $|W_{F_L}| = F_L \cdot \ell = 5 \cdot 10^{-4} \cdot 0{,}25 =$

$= 1{,}25 \cdot 10^{-4}$ Joule

$$Q_{\epsilon\acute{\iota}\sigma o\delta o\varsigma} = I_{\epsilon\pi}^2 R_{o\lambda} \Delta t = (5 \cdot 10^{-3})^2 \cdot 1 \cdot 5 = 125 \cdot 10^{-6} =$$

$= 1{,}25 \cdot 10^{-4}$ Joule.

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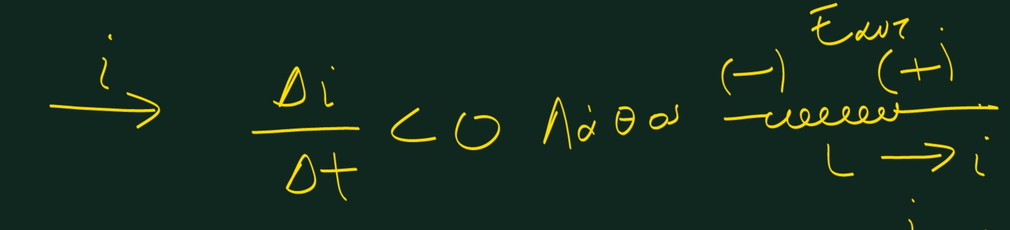
$$E_{αυτ} = -L \cdot \frac{\Delta i}{\Delta t} = -V_L$$

α) Λάθος (δεν υπορεί να έχει σταθερή ένταση).

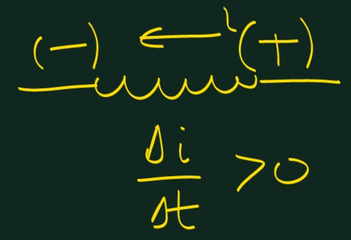
β)

γ) $\xrightarrow{i}$ $\frac{\Delta i}{\Delta t} > 0$ Σωστό.

δ) $\xrightarrow{i}$ $\frac{\Delta i}{\Delta t} < 0$ Λάθος




ε) $\xleftarrow{i}$ $\frac{\Delta i}{\Delta t} > 0$ Λάθος




$\frac{\Delta i}{\Delta t} > 0$

στ) $\xleftarrow{i}$ $\frac{\Delta i}{\Delta t} < 0$ Σωστό.