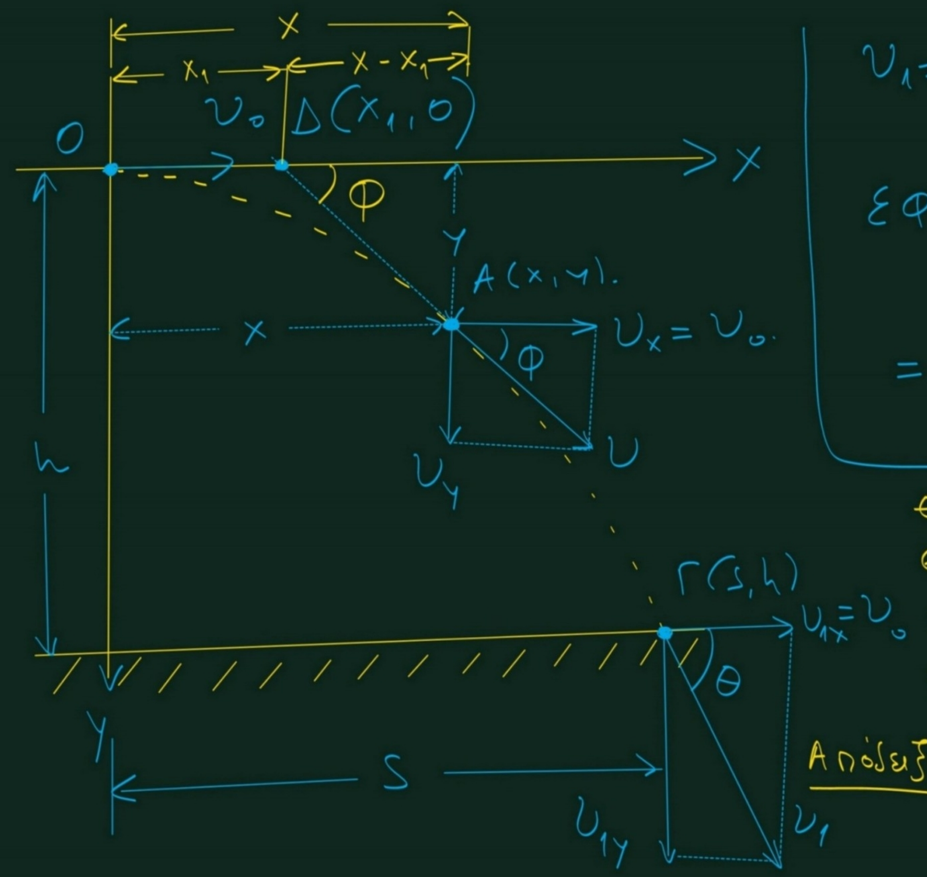


$$\upsilon_1 = \sqrt{\upsilon_0^2 + 2gh}$$

$$\varepsilon\varphi\theta = \frac{\upsilon_{1y}}{\upsilon_{1x}} = \frac{gt_1}{\upsilon_0} =$$

$$= \frac{g}{\upsilon_0} \cdot \sqrt{\frac{2h}{g}} = \sqrt{\frac{2gh}{\upsilon_0^2}}$$

Θεώρημα: Η εφαπτομένη σε τυχαίο σημείο $A(x,y)$ της τροχιάς τέμνει τον άξονα x στο σημείο $\Delta(x_1, 0)$ και ισχύει: $x_1 = \frac{x}{2}$

Απόδειξη: $\varepsilon\varphi\varphi = \frac{\upsilon_y}{\upsilon_x} = \frac{gt}{\upsilon_0} = \frac{gt^2}{\upsilon_0 t} = \frac{2y}{x}$

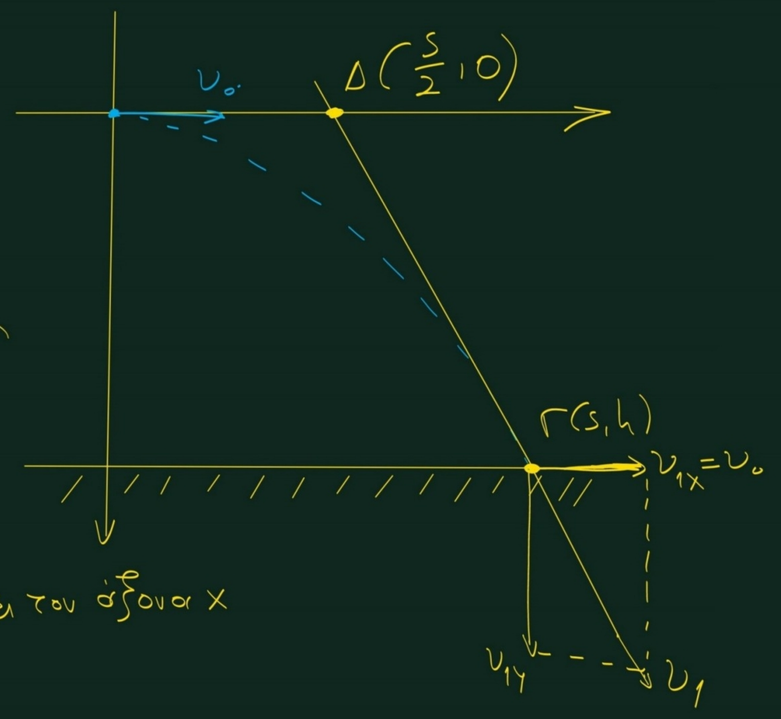
Επίσης: $\varepsilon\varphi\varphi = \dfrac{y}{x - x_1}$

Άρα: $\dfrac{\cancel{y}\,1}{x - x_1} = \dfrac{2\cancel{y}\,1}{x} \Rightarrow$

$$\frac{1}{x - x_1} = \frac{2}{x} \Rightarrow$$

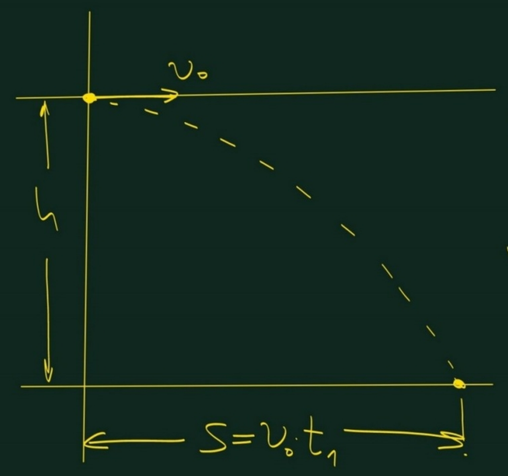
$$x = 2x - 2x_1 \Rightarrow 2x_1 = x \Rightarrow$$

$$x_1 = \frac{x}{2}.$$



Η εφαπτομένη στο σημείο πτώσης $\Gamma(s, h)$ τέμνει τον άξονα x στο σημείο $\Delta\left(\frac{s}{2}, 0\right)$.

Ασκ. 1 σελ. 34



A. $h = \frac{1}{2} g_\Sigma \cdot t_1^2 \Rightarrow 7{,}2 = \frac{1}{2} g_\Sigma \cdot 3^2 \Rightarrow$

$14{,}4 = 9 g_\Sigma \Rightarrow g_\Sigma = 1{,}6 \, m/s^2$

B. i) Ο χρόνος πτώσης είναι $t_1 = \sqrt{\frac{2h}{g_\Sigma}}$ ανεξάρτητος της ταχύτητας v_0.

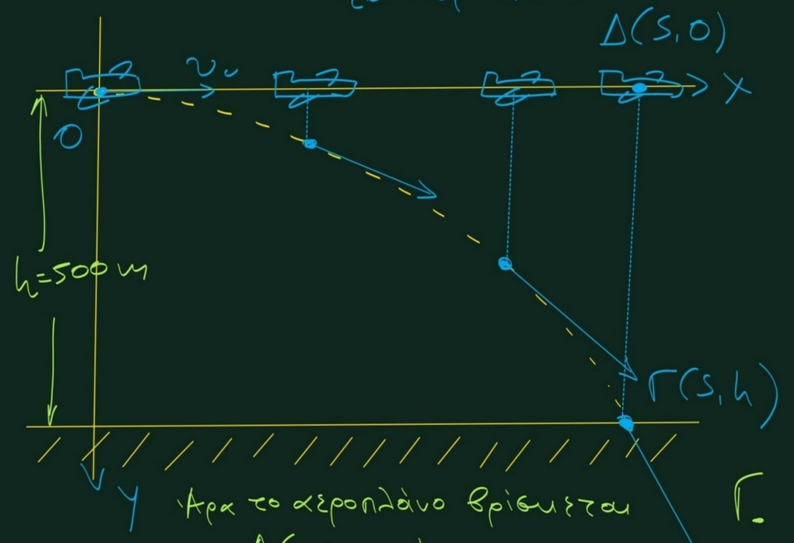
Άρα πάλι: $t_1 = 3 sec.$

ii) $S = v_0 \cdot t_1 = 12 \cdot 3 \Rightarrow$

$S = 36 m.$

Ασκ.2 σελ.34

Η βόμβα βρίσκεται συνεχώς κάτω από το αεροπλάνο.



Άρα το αεροπλάνο βρίσκεται στο Δ(1500,0) και το βλήμα στο Γ(1500,500)

Α. Εξισώσεις ταχύτητας (για τη βόμβα)

$v_x = v_0 \Rightarrow v_x = 150\ m/s.$

$v_y = gt$

Εξισώσεις θέσης.

$x = v_0 t \Rightarrow x = 150t$ (SI)

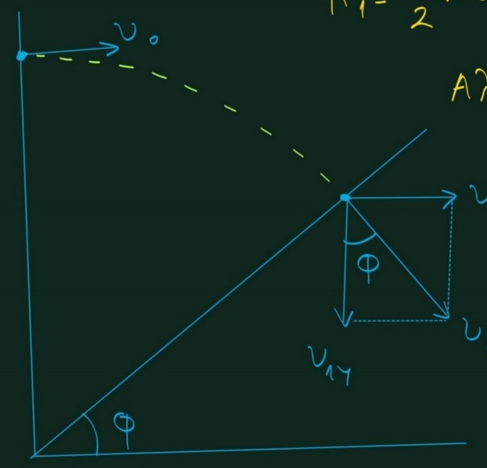
$y = \frac{1}{2} g t^2$

Β. $h = \frac{1}{2} g t_1^2 \Rightarrow 500 = \frac{1}{2} g \cdot 10^2 \Rightarrow$

$1000 = 100g \Rightarrow g = 10\ m/s^2$

Γ. $S = v_0 t_1 = 150 \cdot 10 = 1500\ m$

Ασκ. 1.36



α) $K=\frac{1}{2}m\upsilon_0^2$

$$K_1=\frac{1}{2}m\upsilon_1^2=\frac{1}{2}m(\upsilon_{1x}^2+\upsilon_{1y}^2)=\frac{1}{2}m(\upsilon_0^2+\upsilon_{1y}^2)$$

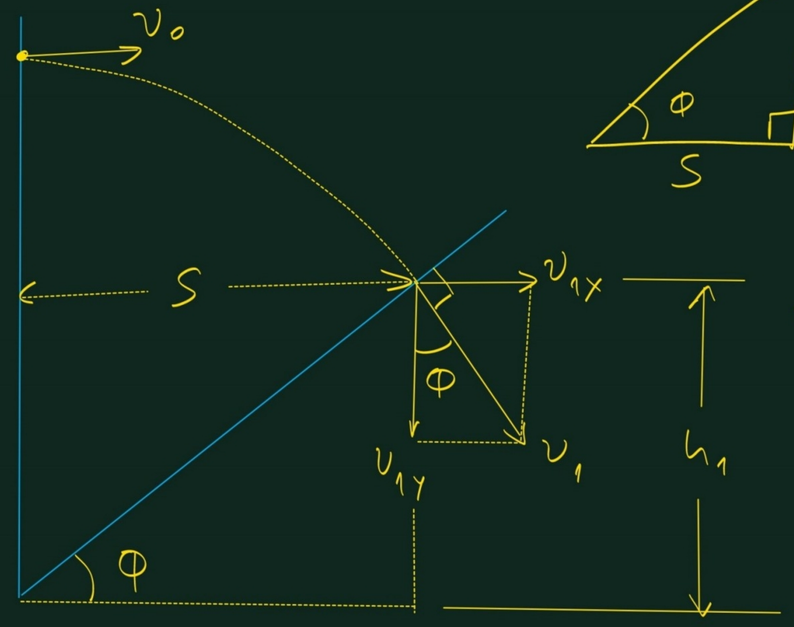
Αλλά: $\varepsilon\varphi\varphi=\frac{\upsilon_{1x}}{\upsilon_{1y}}\Rightarrow\varepsilon\varphi 60^\circ=\frac{\upsilon_0}{\upsilon_{1y}}\Rightarrow$

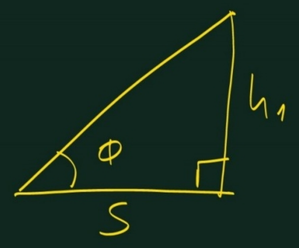
$$\sqrt{3}=\frac{\upsilon_0}{\upsilon_{1y}}\Rightarrow\upsilon_{1y}=\frac{\upsilon_0}{\sqrt{3}}$$

$$K_1=\frac{1}{2}m\left(\upsilon_0^2+\frac{\upsilon_0^2}{3}\right)=\frac{1}{2}m\cdot\frac{4}{3}\upsilon_0^2=\frac{4}{3}\cdot\frac{1}{2}m\upsilon_0^2\Rightarrow$$

$K_1=\frac{4K}{3}$ Σωστό το (i).

β)





$$\varepsilon\varphi\varphi = \frac{h_1}{S} = \frac{h_1}{v_0 t_1}$$

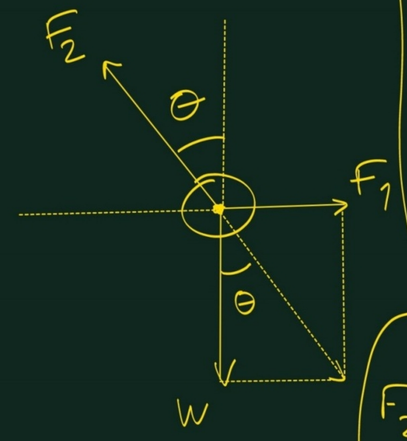
$$\varepsilon\varphi\varphi = \frac{v_{1x}}{v_{1y}} = \frac{v_0}{g t_1}$$

Άρα: $\dfrac{h_1}{v_0 t_1} = \dfrac{v_0}{g t_1} \Rightarrow$

$$\frac{h_1}{v_0} = \frac{v_0}{g} \Rightarrow h_1 = \frac{v_0^2}{g}$$

Σωστό το (iii)

Ασκ. 1.69.

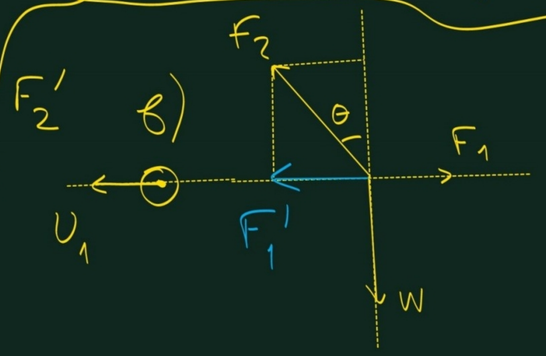


α) $F_2'^2 = F_1^2 + W^2 = 6^2 + 8^2 = 100 \Rightarrow F_2' = 10\,N$

$W = mg = 0{,}8 \cdot 10 = 8\,N$

Άρα: $F_2 = 10\,N$ (γέτρο)

και $\varepsilon\varphi\theta = \dfrac{F_1}{W} = \dfrac{6}{8} = \dfrac{3}{4}$ (κατεύθυνση)



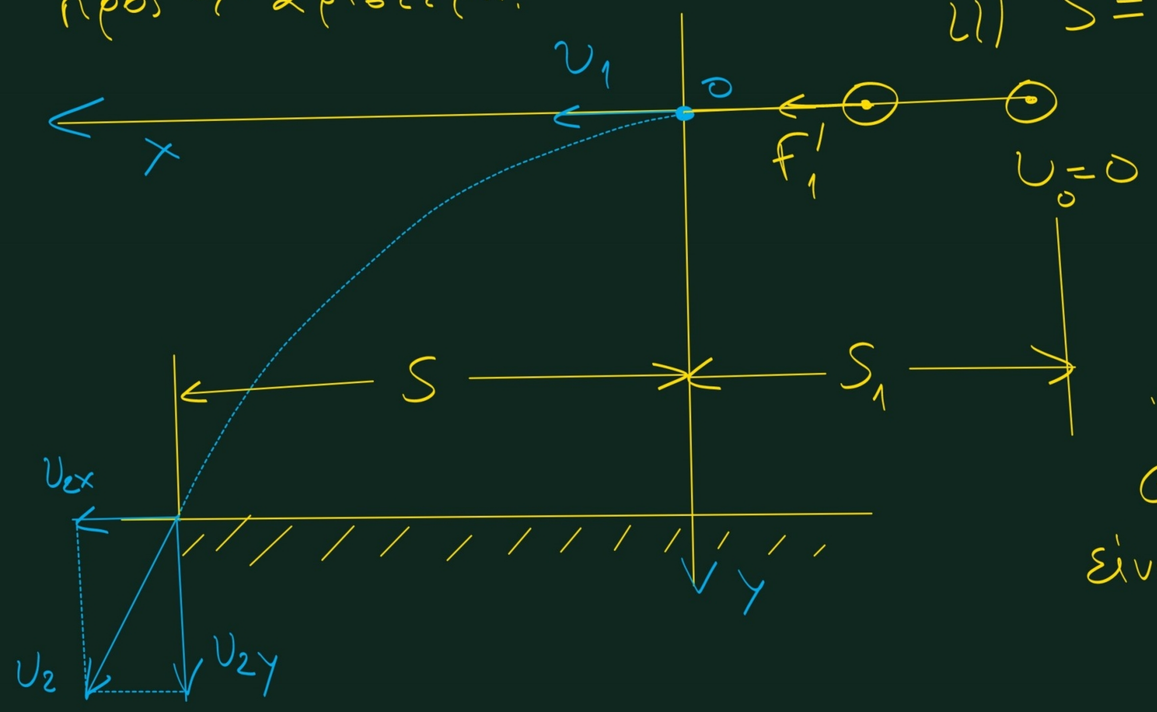
Το σώμα θα κινηθεί υπό την επίδραση της συνισταμένης των $\vec{F_2}$ και $\vec{W}$ που είναι αντίθετη της $\vec{F_1}$

$F_1' = ma \Rightarrow a = \dfrac{F_1'}{m}$

Άρα: $F_1' = 6\,N$

$U_1 = at_1 = \dfrac{F_1'}{m} \cdot t_1 = \dfrac{6}{0{,}8} \cdot 4 = \dfrac{6}{0{,}2} = 30\,m/s$

γ) Τη χρονική στιγμή $t_1 = 4\,sec$ το σώμα εκτελεί οριζόντια βολή με αρχική ταχύτητα $\upsilon_1 = 30\,m/s$ προς τ' αριστερά.



i) $h = \frac{1}{2} g t_2^2 \Rightarrow$

$45 = \frac{1}{2} \cdot 10 \cdot t_2^2 \Rightarrow 90 = 10 t_2^2 \Rightarrow$

$t_2^2 = 9 \Rightarrow t_2 = 3\,sec$

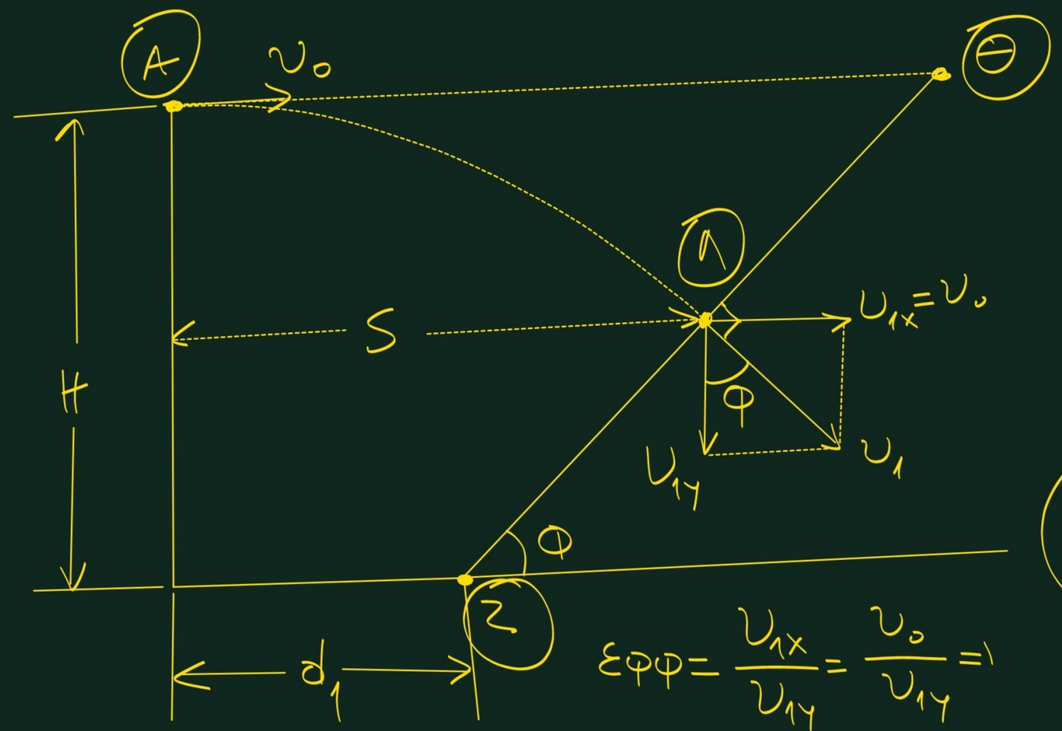
ii) $S = \upsilon_1 \cdot t_2 = 30 \cdot 3 = 90\,m.$

$S_1 = \frac{1}{2} a t_1^2 = \frac{1}{2} \cdot \frac{F_1'}{m} \cdot t_1^2 =$

$= \frac{1}{2} \cdot \frac{6}{0,8} \cdot 4^2 = \frac{6 \cdot 16}{1,6} = 60\,m$

Άρα η συνολική οριζόντια απόσταση από τη στιγμή $t=0$ μέχρι το έδαφος είναι: $S_{ολ} = S + S_1 = 150\,m$

Ασκ. 1.73



$$\varepsilon\varphi\varphi = \frac{v_{1x}}{v_{1y}} = \frac{v_0}{v_{1y}} \Rightarrow$$

$$\frac{\eta\mu\varphi}{\sigma\upsilon\nu\varphi} = \frac{v_0}{v_{1y}} \Rightarrow \frac{0,6}{0,8} = \frac{22,5}{v_{1y}} \Rightarrow 0,6\,v_{1y} = \cancel{20}^{\,18} \Rightarrow v_{1y} = \frac{\cancel{20}^{\,18}}{0,6} = \frac{\cancel{200}^{\,180}}{6} = \frac{\cancel{100}^{\,90}}{3}\,m/s \Rightarrow$$

$$v_{1y} = 30\,m/s$$

α) $v_{1y} = g t_1 \Rightarrow v_0 \sigma\upsilon\nu\varphi = g t_1$

$$S = v_0 t_1 \Rightarrow t_1 = \frac{S}{v_0}.$$

$$t_1 = \frac{v_0 \sigma\upsilon\nu\varphi}{g} = \frac{22,5 \cdot 0,8}{10} = \frac{20}{10} \Rightarrow$$

$$t_1 = 2\,sec$$

$$K_1 = \frac{1}{2} m v_1^2 = \frac{1}{2} m (v_{1x}^2 + v_{1y}^2)$$

$$\varepsilon\varphi\varphi = \frac{v_{1x}}{v_{1y}} \Rightarrow \varepsilon\varphi\varphi = \frac{v_0}{v_{1y}} \Rightarrow$$

$$v_{1y} = \frac{v_0}{\varepsilon\varphi\varphi} = \frac{v_0}{\frac{\eta\varphi}{\sigma\upsilon\varphi}} = \frac{v_0}{\frac{0,6}{0,8}} = \frac{v_0}{\frac{3}{4}} = \frac{4v_0}{3}$$

Άρα: $K_1 = \frac{1}{2}m\left(v_0^2 + \left(\frac{4v_0}{3}\right)^2\right) =$

$$= \frac{1}{2}\cdot m\cdot\left(v_0^2 + \frac{16v_0^2}{9}\right) =$$

$$= \frac{1}{2}m\cdot\frac{25v_0^2}{9} = \frac{1}{2}mv_0^2\cdot\frac{25}{9} = \frac{1}{2}\cdot 0,32\cdot\frac{90^2}{16}\cdot\frac{25}{9} = \frac{\overset{1}{\cancel{0,32}}\cdot\overset{10}{\cancel{90}}\cdot 90\cdot 25}{\underset{1}{\cancel{32}}\cdot\cancel{9}}$$

$$= \frac{\overset{100}{900}}{4} = 225\,\text{Joule}$$

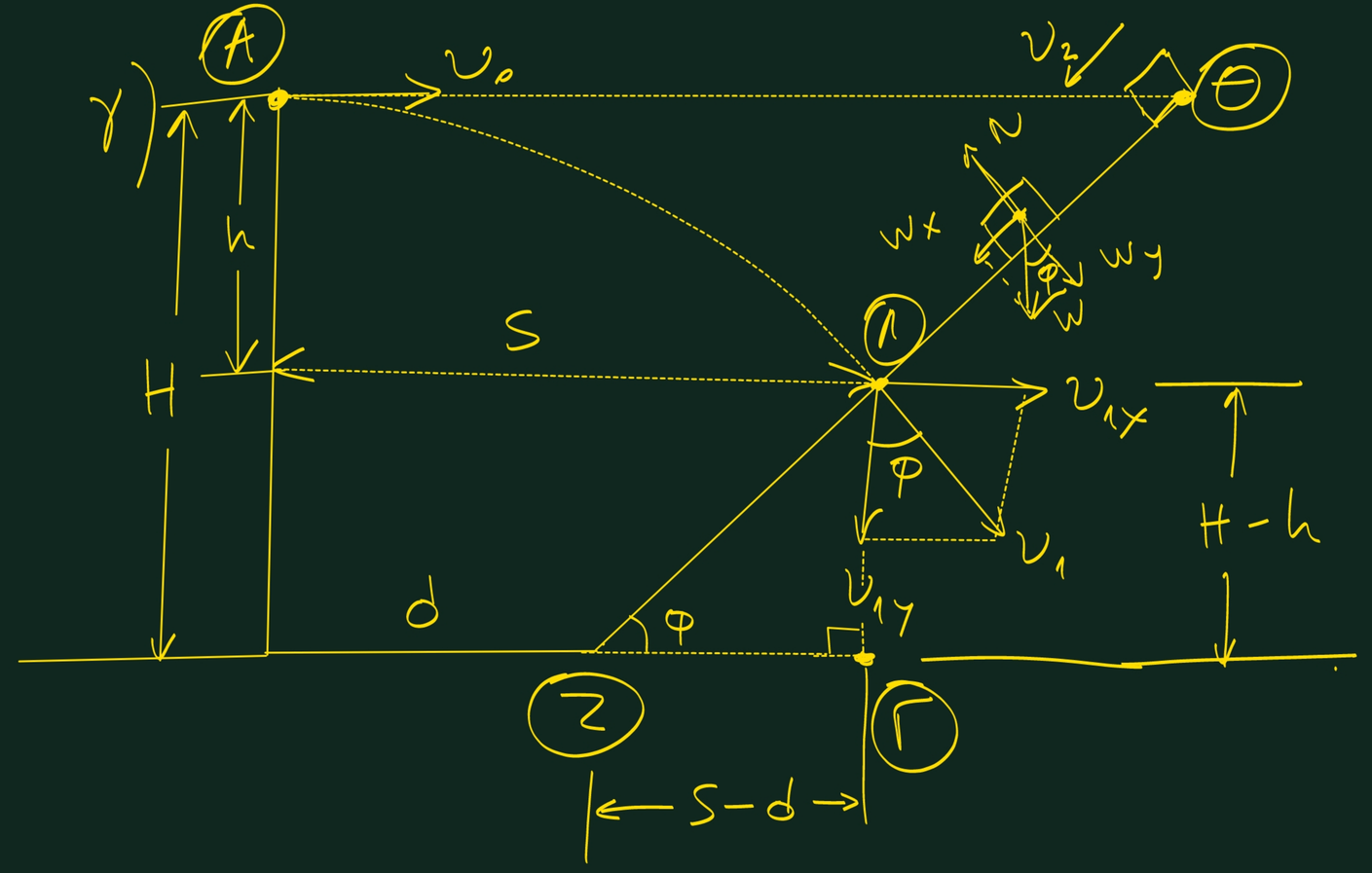
β) Από το (α) έχουμε:

$$\upsilon_{1y} = 30\,m/s$$

Άρα: $\upsilon_{1y} = g t_1 \Rightarrow$

$$30 = 10 t_1 \Rightarrow$$

$$t_1 = 3\,sec$$



γ) $S = \upsilon_0 t_1 = 22{,}5 \cdot 3 = 67{,}5\,m$

$$h = \frac{1}{2} g t_1^2 = \frac{1}{2} \cdot 10 \cdot 3^2 = 45\,m$$

Από το τρίγωνο $\overset{\Delta}{ZΛΓ}$ έχουμε: $εφφ = \frac{ΛΓ}{ZΓ} \Rightarrow \frac{w_y φ}{συνφ} = \frac{H-h}{S-d} \Rightarrow \frac{\cancel{0{,}6}^{\,3}}{\cancel{0{,}8}_{\,4}} = \frac{H-45}{67{,}5-47{,}5} \Rightarrow$

$$\frac{3}{\cancel{4}_{1}} = \frac{H-45}{\cancel{20}_{5}} \Rightarrow$$

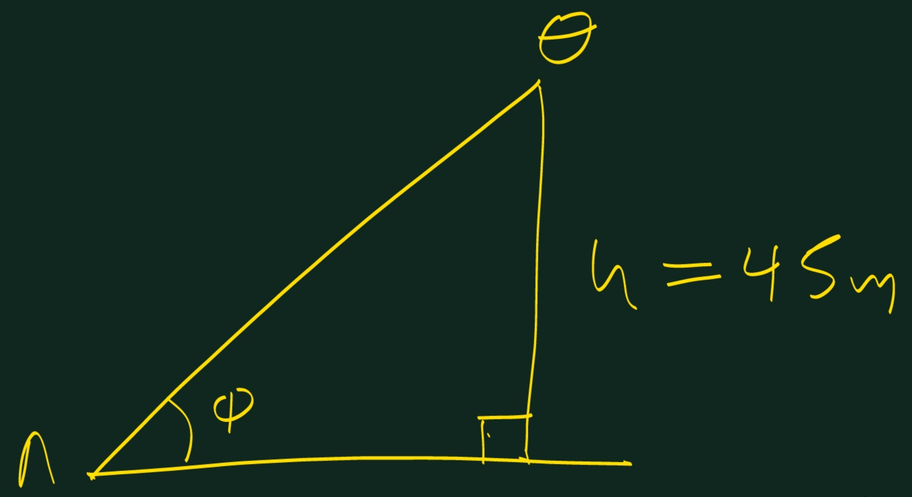
$$15 = H - 45 \Rightarrow H = 60\,m$$

δ) Για το σώμα που ξεκινά από το Θ έχουμε:

$$\Theta\Lambda = \upsilon_2 \cdot t_1 + \frac{1}{2} a \cdot t_1^2$$

$$\Sigma F_x = ma \Rightarrow \cancel{m} g\,\eta\mu\varphi = \cancel{m} a \Rightarrow$$

$$a = g\,\eta\mu\varphi = 10 \cdot 0{,}6 = 6\,m/s^2$$



$$\eta\mu\varphi = \frac{h}{\Theta\Lambda} \Rightarrow 0{,}6 = \frac{45}{\Theta\Lambda} \Rightarrow$$

$$\Theta\Lambda = \frac{45}{0{,}6} = \frac{15}{0{,}2} = 75\,m.$$

Άρα: $75 = \upsilon_2 \cdot 3 + \frac{1}{2} \cdot 6 \cdot 3^2 \Rightarrow$

$$75 = 3\upsilon_2 + 27 \Rightarrow$$

$$25 = \upsilon_2 + 9 \Rightarrow \upsilon_2 = 16\,m/s$$