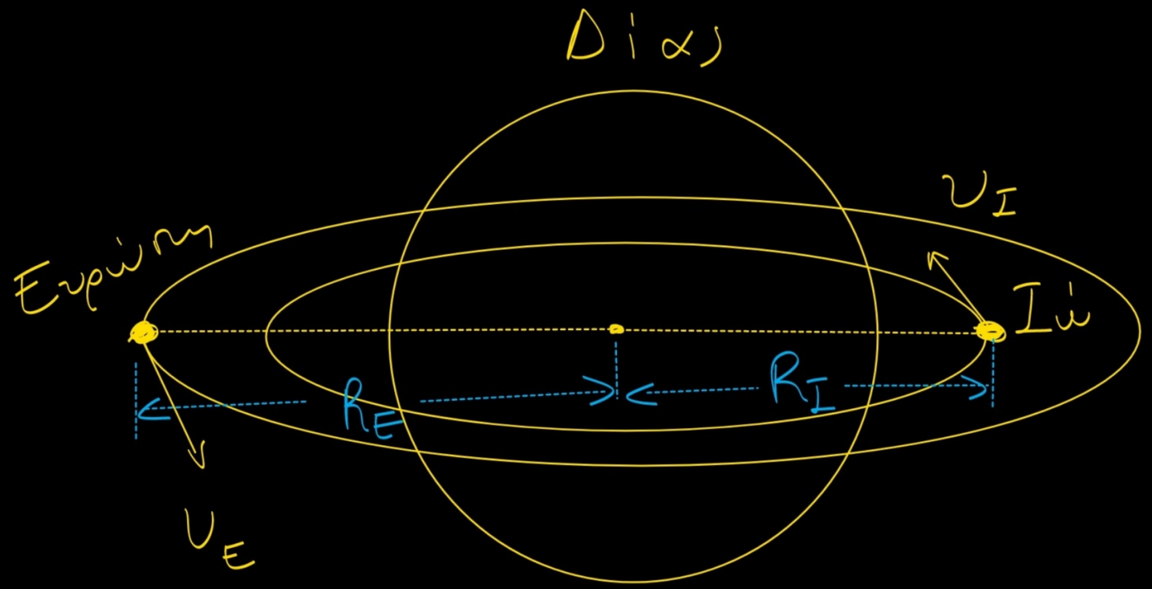
Θέμα 4) (Ιώ και Ευρώπη).



4.1) $R_I = 432 \cdot 10^3 \text{ km} = 432 \cdot 10^6 \text{ m}$

$R_E = 675 \cdot 10^3 \text{ km}$

$T_I = 1,57 \text{ days}.$

$$v_I = \frac{2\pi R_I}{T_I} = \frac{2 \cdot 3,14 \cdot 432 \cdot 10^6}{1,57 \cdot 24 \cdot 3600} =$$

$$= \frac{4 \cdot 432 \cdot 10^6}{24 \cdot 36 \cdot 10^2} \Rightarrow v_I = 2 \cdot 10^4 \text{ m/s}.$$

4.2) $$m_I \cdot \frac{v_I^2}{R_I} = G \frac{M_\Delta \cdot m_I}{R_I^2}$$

$F_{\text{κεντρομόλος}} = F_{\text{βαρύτητας}}.$

$$\cancel{m_I} \cdot \frac{v_I^2}{\cancel{R_I}} = G \frac{M_\Delta \cdot \cancel{m_I}}{R_I^2} \Rightarrow$$

$$v_I^2 = \frac{GM_\Delta}{R_I} \Rightarrow M_\Delta = \frac{R_I \cdot v_I^2}{G} \Rightarrow$$

$$M_\Delta = \frac{432 \cdot 10^6 \cdot 4 \cdot 10^8}{6{,}67 \cdot 10^{-11}} \simeq \frac{4 \cdot 432 \cdot 10^{14}}{\frac{20}{3} \cdot 10^{-11}} =$$

$$= \frac{12 \cdot 432}{20} \cdot 10^{25} \, kg.$$

$$G = 6{,}67 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}.$$

$$F = G \frac{m_1 m_2}{r^2} \Rightarrow$$

$$kg \cdot \frac{m}{s^2} = G \cdot \frac{kg^2}{m^2} \Rightarrow$$

$$G = \frac{kg \cdot m^3}{kg^2 \cdot s^2} \Rightarrow$$

$$G = \frac{m^3}{kg \cdot s^2}$$

4.3 Τρίτος Νόμος Kepler

$$m\frac{U^2}{r} = G\frac{Mm}{r^2} \Rightarrow$$

$$U^2 = \frac{GM}{r} \Rightarrow$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r} \Rightarrow$$

$$r^3 = \frac{GM}{4\pi^2}\cdot T^2$$

Δηλαδή οι κύβοι των ακτίνων είναι ανάλογοι με τα τετράγωνα των περιόδων.

$$\frac{R_I^3}{T_I^2} = \frac{R_E^3}{T_E^2} \Rightarrow$$

$$T_E^2 = T_I^2 \cdot \frac{R_E^3}{R_I^3} \Rightarrow T_E = T_I \cdot \left(\frac{R_E}{R_I}\right)^{3/2} \Rightarrow$$

$$T_E = T_I \cdot \left(\frac{675}{432}\right)^{3/2} = T_I \cdot \left(\frac{25\cdot 27}{2\cdot 6\cdot 36}\right)^{3/2}$$

$$675 = 625 + 50 =$$

$$= 25\cdot 25 + 25\cdot 2 = 25\cdot 27$$

$$= T_I \cdot \left(\frac{25}{16}\right)^{3/2}$$

$$T_E = T_I \cdot \left(\frac{25}{16}\right)^{3/2} \Rightarrow$$

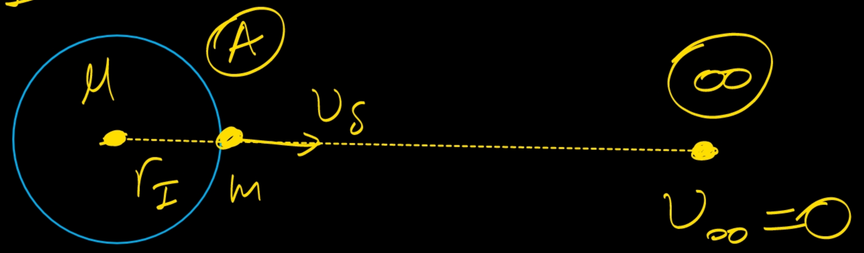
$$T_E = T_I \cdot \sqrt{\left(\frac{25}{16}\right)^3} = T_I \cdot \left(\frac{5}{4}\right)^3 =$$

$$= T_I \cdot \frac{125}{16} = 1{,}57 \cdot \frac{125}{16} \text{ days}.$$

4.4.

$I\omega$.

$M = M_I = 9 \cdot 10^{22}$ kg.



$$K_A + U_A = K_\infty + U_\infty \Rightarrow$$

$$\frac{1}{2} m v_s^2 - \frac{G M m}{r_I} = 0 \Rightarrow$$

$$v_s^2 = \frac{2GM}{r_I} = \frac{2 \cdot 6{,}67 \cdot 10^{-11} \cdot 9 \cdot 10^{22}}{1{,}8 \cdot 10^6} \Rightarrow$$

$$v_s^2 = \frac{18 \cdot 6{,}67}{1{,}8 \cdot 10^6} \cdot 10^{11} = \frac{18 \cdot 6{,}67}{1{,}8} \cdot 10^5 = \frac{18 \cdot 6{,}67}{18} \cdot 10^6 = 6{,}67 \cdot 10^6 \Rightarrow$$

$$v_s = \sqrt{6{,}67 \cdot 10^3} \Rightarrow v_s = 2{,}58 \cdot 10^3 \text{ m/s}.$$