$$V_A = \frac{W_{A\to\infty}}{m}$$

$$V_A - V_B = \frac{W_{A\to B}}{m}$$

Αρα: $W_{F(A\to B)} = m(V_A - V_B)$

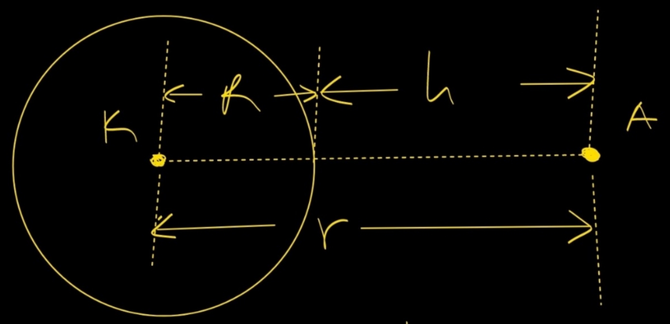
$$V = -\frac{GM}{r}$$

$$U = mV = -\frac{GMm}{r}$$

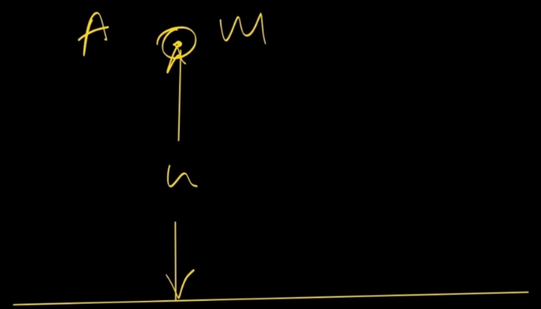
$$g = \frac{F}{m} = \frac{\frac{GMm}{r^2}}{m} = \frac{GM}{r^2}$$

$\vec{g}$ = έντασα

$\vec{g} \uparrow\uparrow \vec{F}$ (αφού $m>0$)



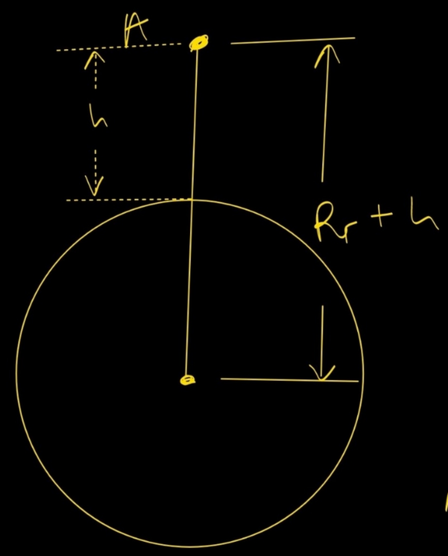
$r = R + h$



$$U_A = mgh.$$

Αν $x \ll 1$

τότε $\frac{1}{1+x} \simeq 1-x$



$$U_A = -\frac{GM_\gamma}{R_\gamma + h}$$

$$GM_\gamma = g_0 R_\gamma^2$$

$$U_A = -\frac{mg_0 R_\gamma^2}{R_\gamma + h} =$$

$$= -\frac{mg_0 R_\gamma^2}{R_\gamma\left(1+\frac{h}{R_\gamma}\right)} = -\frac{mg_0 R_\gamma}{1+\frac{h}{R_\gamma}}$$

Αν $h \ll R_\gamma$ τότε

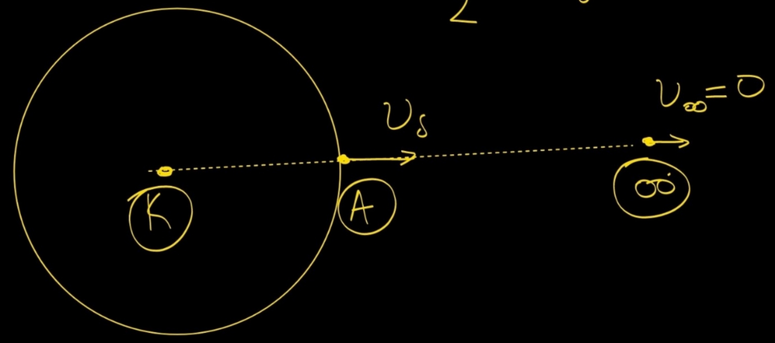
$$\frac{1}{1+\frac{h}{R_\gamma}} \simeq 1-\frac{h}{R_\gamma}$$

$$U_A \simeq -mg_0 R_\gamma\left(1-\frac{h}{R_\gamma}\right) = -mg_0 R_\gamma + mg_0 h$$

Ταχύτητα διαφυγής.

ΘΜΚΕ: $A \to \infty$: $\cancel{\frac{1}{2}mU_\infty^2} - \frac{1}{2}mU_s^2 = m(V_A - \cancel{V_\infty}) \Rightarrow$

$$-\frac{1}{2}mU_s^2 = -m\cdot\frac{GM_\Gamma}{R_\Gamma} \Rightarrow \quad U_s^2 = \frac{2GM_\Gamma}{R_\Gamma} \Rightarrow$$



$$U_s^2 = \frac{2g_0R_\Gamma^2}{R_\Gamma} = 2g_0R_\Gamma \Rightarrow$$

$$U_s = \sqrt{2g_0R_\Gamma}$$

$$U_s = \sqrt{2\cdot 10\cdot 6{,}4\cdot 10^6} = \sqrt{2}\cdot 8\cdot 10^3 \Rightarrow$$

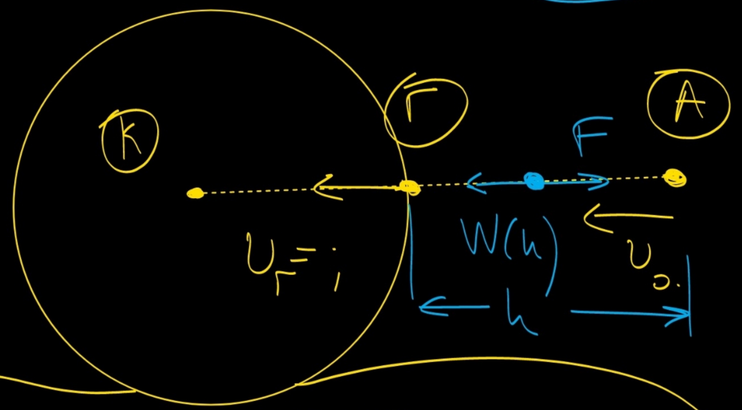
$$U_s = 8\sqrt{2}\cdot 10^3 \ m/s$$

Ασκ. 104 §2.203.

$h = R_r$

$v_0 = 8 \cdot 10^3$ m/s.

$m = 8 \cdot 10^3$ kg.



α) ΑΔΕ: Α → Γ

$K_A + U_A = K_r + U_r \Rightarrow$

$$\frac{1}{2}mv_0^2 - G\frac{M_r \cdot m}{R_r + h} = \frac{1}{2}mv_r^2 - G\frac{M_r \cdot m}{R_r} \Rightarrow$$

$$v_0^2 - \frac{2GM_r}{2R_r} = v_r^2 - \frac{2GM_r}{R_r} \Rightarrow$$

$$v_0^2 - \frac{g_0 R_r^2}{R_r} = v_r^2 - \frac{2g_0 R_r^2}{R_r} \Rightarrow$$

$$v_0^2 - g_0 R_r = v_r^2 - 2g_0 R_r \Rightarrow v_r^2 = v_0^2 + g_0 R_r \Rightarrow$$

$$v_r^2 = (8 \cdot 10^3)^2 + 10 \cdot 6{,}4 \cdot 10^6 = (8 \cdot 10^3)^2 + (8 \cdot 10^3)^2 = 2 \cdot (8 \cdot 10^3)^2$$

Άρα: $v_r = \sqrt{2 \cdot (8 \cdot 10^3)^2} = 8 \cdot 10^3 \sqrt{2}$ m/s.

β) ΘΜΚΕ: Α → Γ:

$\Delta K = \Sigma W \Rightarrow$

$K_\Gamma - K_A = m(V_A - V_\Gamma) + W_F \Rightarrow$

$$0 - \frac{1}{2}mv_0^2 = m \cdot \left(-\frac{GM_\Gamma}{R_\Gamma + h} - \left(-\frac{GM_\Gamma}{R_\Gamma} \right) \right) - F \cdot h \Rightarrow$$

$$-\frac{1}{2}mv_0^2 = -\frac{mg_0R_\Gamma^2}{2R_\Gamma} + \frac{mg_0R_\Gamma^2}{R_\Gamma} - F \cdot h \Rightarrow$$

$$-\frac{1}{2}mv_0^2 = -\frac{mg_0R_\Gamma}{2} + mg_0R_\Gamma - F \cdot h \Rightarrow F \cdot h = \frac{mg_0R_\Gamma}{2} + \frac{mv_0^2}{2} \Rightarrow$$

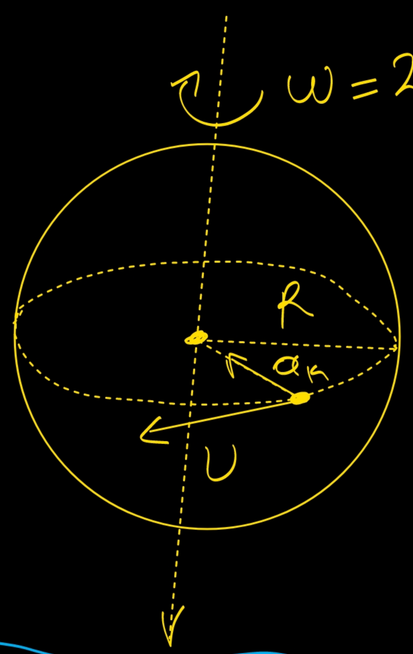
$$F = \frac{mg_0}{2} + \frac{mv_0^2}{2R_\Gamma} = 40.000 + \frac{8 \cdot 10^3 \cdot 64 \cdot 10^6}{2 \cdot 6,4 \cdot 10^6} = 8 \cdot 10^4 N$$

Θέμα 4 (Αστέρας Νετρονίων)

$f = 700\ Hz$ = 700 περιστροφές το δευτερόλεπτο.

$R = 10\ km$

$GM = 2 \cdot 10^{20}\ N \cdot m^2/kg$.



4.1. $\upsilon = \omega R = 2\pi f R =$

$= 2\pi \cdot 7 \cdot 10^2 \cdot 10 \cdot 10^3 =$

$= 14\pi \cdot 10^7\ m/sec$

4.2

$$a_K = \frac{\upsilon^2}{R} = \frac{(14\pi \cdot 10^7)^2}{10 \cdot 10^3} \Rightarrow$$

$$a_K = \frac{196\pi^2 \cdot 10^{14}}{10^4} = 196\pi^2 \cdot 10^{10} \Rightarrow$$

$$a_K = 196 \cdot 10^{11}\ m/sec^2 = 1,96 \cdot 10^{13}\ m/s^2$$

4.3) $g = \frac{GM}{R^2} \Rightarrow$

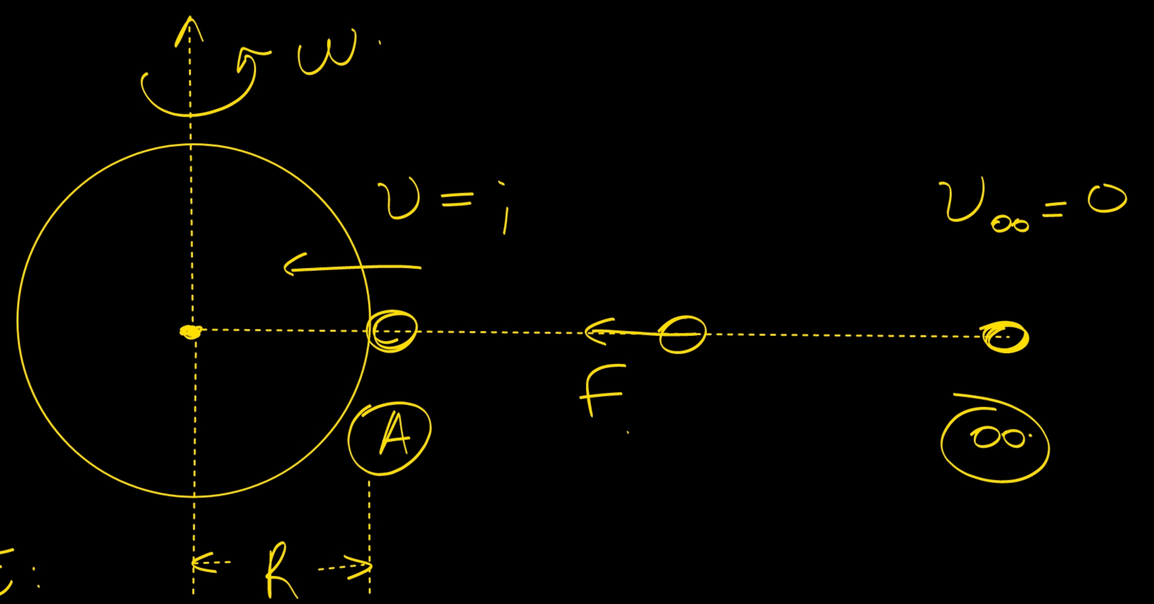
$$g = \frac{2 \cdot 10^{20}}{(10^4)^2} \Rightarrow$$

$$g = \frac{2 \cdot 10^{20}}{10^8} \Rightarrow$$

$$g = 2 \cdot 10^{12}\ m/s^2$$

$g \simeq 2 \cdot 10^{11}\ g$.

4.4.)



ADE:

$$K_\infty + U_\infty = K_A + U_A \Rightarrow$$

$$0 + 0 = \frac{1}{2}mv^2 - \frac{GMm}{R^2} \Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{R^2} \Rightarrow v^2 = \frac{2GM}{R^2} \Rightarrow$$

$$v^2 = \frac{4\cdot 10^{20}}{(10^4)^2} = \frac{4\cdot 10^{20}}{10^8} = 4\cdot 10^{12} \Rightarrow$$

$$v = 2\cdot 10^6 \text{ m/s}.$$

$$c = 3\cdot 10^8 \text{ m/s}.$$

$$\frac{v}{c} \simeq \frac{1}{150}$$