$$V_A = \frac{W_{A\to\infty}}{m}$$

$$V_A - V_B = \frac{W_{A\to B}}{m}$$

Άρα: $W_{F(A\to B)} = m(V_A - V_B)$

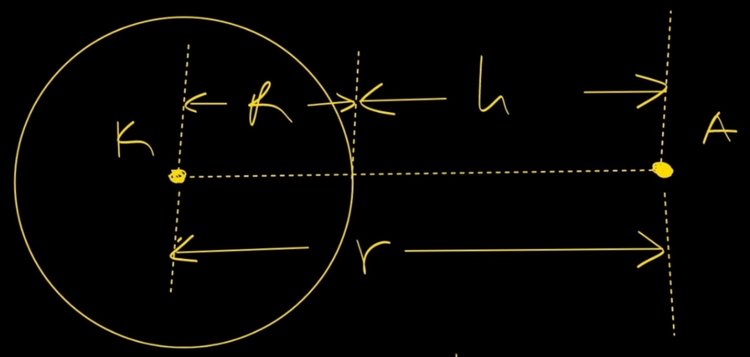
$$V = -\frac{GM}{r}$$

$$U = mV = -\frac{GMm}{r}$$

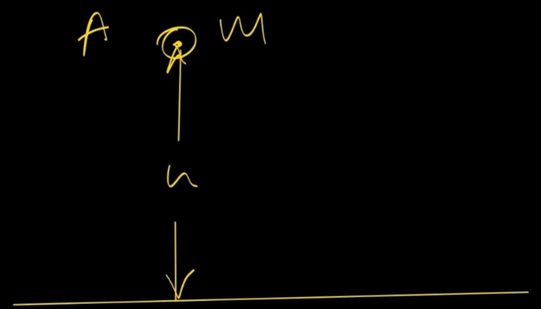
$$g = \frac{F}{m} = \frac{\frac{GMm}{r^2}}{m} = \frac{GM}{r^2}$$

$\vec{g}$ = ένταση

$\vec{g} \uparrow\uparrow \vec{F}$ (αφού $m>0$)



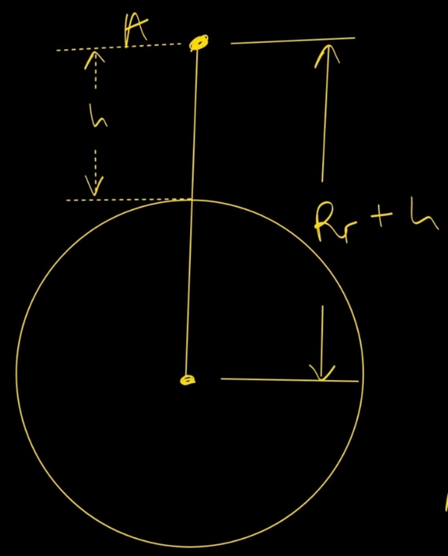
$r = R + h$



$$U_A = mgh.$$

Αν $x \ll 1$

τότε $\frac{1}{1+x} \simeq 1-x$



$$U_A = -\frac{GM_\gamma}{R_\gamma + h}$$

$$GM_\gamma = g_0 R_\gamma^2$$

$$U_A = -\frac{mg_0 R_\gamma^2}{R_\gamma + h} =$$

$$= -\frac{mg_0 R_\gamma^2}{R_\gamma\left(1+\frac{h}{R_\gamma}\right)} = -\frac{mg_0 R_\gamma}{1+\frac{h}{R_\gamma}}$$

Αν $h \ll R_\gamma$ τότε

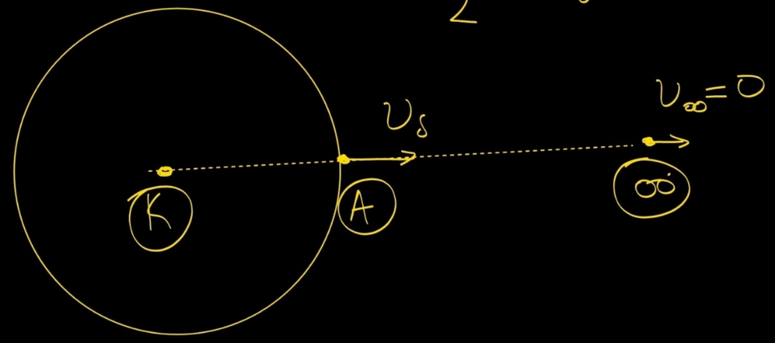
$$\frac{1}{1+\frac{h}{R_\gamma}} \simeq 1-\frac{h}{R_\gamma}$$

$$U_A \simeq -mg_0 R_\gamma\left(1-\frac{h}{R_\gamma}\right) = -mg_0 R_\gamma + mg_0 h$$

Ταχύτητα διαφυγής.

ΘΜΚΕ: $A \to \infty$: $\cancel{\frac{1}{2}mU_\infty^2} - \frac{1}{2}mU_s^2 = m(V_A - \cancel{V_\infty}) \Rightarrow$

$$-\frac{1}{2}mU_s^2 = -m\cdot\frac{GM_\Gamma}{R_\Gamma} \Rightarrow \quad U_s^2 = \frac{2GM_\Gamma}{R_\Gamma} \Rightarrow$$



$$U_s^2 = \frac{2g_0R_\Gamma^2}{R_\Gamma} = 2g_0R_\Gamma \Rightarrow$$

$$U_s = \sqrt{2g_0R_\Gamma}$$

$$U_s = \sqrt{2\cdot 10\cdot 6{,}4\cdot 10^6} = \sqrt{2}\cdot 8\cdot 10^3 \Rightarrow$$

$$U_s = 8\sqrt{2}\cdot 10^3 \ m/s$$

Ασκ. 104 σελ. 203.

$h = R_\Gamma$

$v_0 = 8 \cdot 10^3$ m/s.

$m = 8 \cdot 10^3$ kg.

α) ΑΔΕ: Α → Γ

$K_A + U_A = K_\Gamma + U_\Gamma \Rightarrow$

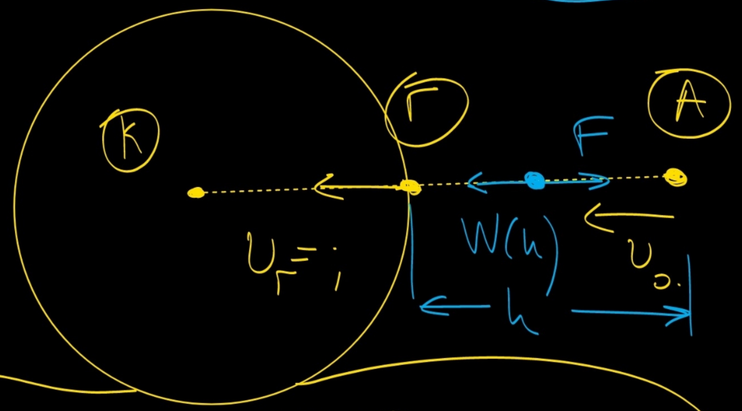
$$\frac{1}{2} m v_0^2 - G\frac{M_\Gamma \cdot m}{R_\Gamma + h} = \frac{1}{2} m v_\Gamma^2 - G\frac{M_\Gamma \cdot m}{R_\Gamma} \Rightarrow$$

$$v_0^2 - \frac{2GM_\Gamma}{2R_\Gamma} = v_\Gamma^2 - \frac{2GM_\Gamma}{R_\Gamma} \Rightarrow$$

$$v_0^2 - \frac{g_0 R_\Gamma^2}{R_\Gamma} = v_\Gamma^2 - \frac{2g_0 R_\Gamma^2}{R_\Gamma} \Rightarrow$$

$$v_0^2 - g_0 R_\Gamma = v_\Gamma^2 - 2g_0 R_\Gamma \Rightarrow v_\Gamma^2 = v_0^2 + g_0 R_\Gamma \Rightarrow$$

$$v_\Gamma^2 = (8\cdot 10^3)^2 + 10 \cdot 6{,}4 \cdot 10^6 = (8\cdot 10^3)^2 + (8\cdot 10^3)^2 = 2\cdot(8\cdot 10^3)^2$$



Αρα: $v_\Gamma = \sqrt{2\cdot(8\cdot 10^3)^2} = 8\cdot 10^3\sqrt{2}$ m/s.

β) ΘΜΚΕ: Α → Γ:

$\Delta K = \Sigma W \Rightarrow$

$K_\Gamma - K_A = m(V_A - V_\Gamma) + W_F \Rightarrow$

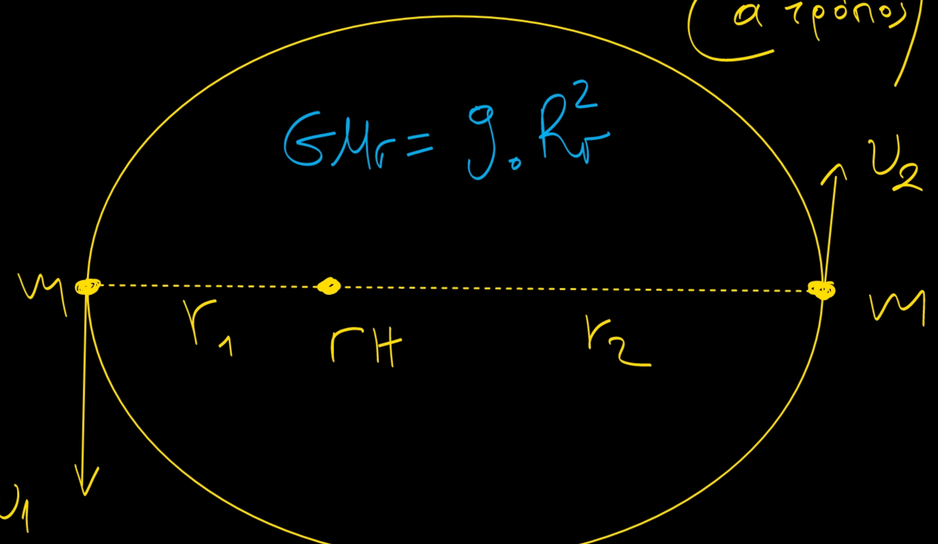
$$0 - \frac{1}{2}mv_0^2 = m \cdot \left(-\frac{GM_\Gamma}{R_\Gamma + h} - \left(-\frac{GM_\Gamma}{R_\Gamma} \right) \right) - F \cdot h \Rightarrow$$

$$-\frac{1}{2}mv_0^2 = -\frac{mg_0R_\Gamma^2}{2R_\Gamma} + \frac{mg_0R_\Gamma^2}{R_\Gamma} - F \cdot h \Rightarrow$$

$$-\frac{1}{2}mv_0^2 = -\frac{mg_0R_\Gamma}{2} + mg_0R_\Gamma - F \cdot h \Rightarrow F \cdot h = \frac{mg_0R_\Gamma}{2} + \frac{mv_0^2}{2} \Rightarrow$$

$$F = \frac{mg_0}{2} + \frac{mv_0^2}{2R_\Gamma} = 40.000 + \frac{8 \cdot 10^3 \cdot 64 \cdot 10^6}{2 \cdot 6,4 \cdot 10^6} = 8 \cdot 10^4 N$$

Ασκ. 105) σελ. 203.)



β' τρόπος (διατήρηση ενέργειας)

$K_1 + U_1 = K_2 + U_2 \Rightarrow$

$$\frac{1}{2}mv_1^2 - \frac{GM_\Gamma m}{r_1} = \frac{1}{2}mv_2^2 - \frac{GM_\Gamma m}{r_2} \Rightarrow v_1^2 - \frac{2GM_\Gamma}{r_1} = v_2^2 - \frac{2GM_\Gamma}{r_2} \Rightarrow v_2^2 = v_1^2 - 2g_0R_\Gamma^2\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \Rightarrow$$

(α' τρόπος)

α) Από τη διατήρηση της στροφορμής έχουμε:

$$L_1 = L_2 \Rightarrow mv_1r_1 = mv_2r_2 \Rightarrow$$

$$v_1r_1 = v_2r_2 \Rightarrow$$

$$8\cdot10^3\cdot7\cdot10^6 = v_2\cdot9\cdot10^6 \Rightarrow$$

$$v_2 = \frac{8\cdot7}{9}\cdot10^3 = \frac{54+2}{9}\cdot10^3 \Rightarrow$$

$$v_2 = \left(6+\frac{2}{9}\right)\cdot10^3 = 6{,}22\cdot10^3\ m/s.$$

β) $K + U = \text{σταθ} \Rightarrow$

$$\frac{1}{2}mv^2 - \frac{GMr}{r} = \text{σταθ}$$

σε όλη την ελλειπτική τροχιά.

Άρα: $E_1 + W = E_\infty \Rightarrow$

$$K_1 + U_1 + W = \cancel{K_\infty}^{0} + \cancel{U_\infty}^{0} \Rightarrow$$

$$\frac{1}{2}mv_1^2 - \frac{GMr\,m}{r_1} + W = 0$$

$$v_2^2 = v_1^2 - 2g_0R^2\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \Rightarrow v_2^2 = v_1^2 - 2g_0R^2\left(\frac{r_2 - r_1}{r_1 \cdot r_2}\right)$$

$$v_2^2 = 64\cdot 10^6 - 2\cdot 10\cdot (6{,}4\cdot 10^6)^2 \cdot \frac{9\cdot 10^6 - 7\cdot 10^6}{7\cdot 9\cdot 10^{12}} \Rightarrow$$

$$v_2^2 = 64\cdot 10^6 - 2\cdot 10\cdot 6{,}4^2\cdot 10^{12}\cdot \frac{2\cdot 10^6}{7\cdot 9\cdot 10^{12}} \Rightarrow$$

$$v_2^2 = 64\cdot 10^6 - \frac{2\cdot 10\cdot 6{,}4^2\cdot 2\cdot 10^6}{7\cdot 9} \Rightarrow$$

$$v_2^2 = 64\cdot 10^6 - \frac{40\cdot 6{,}4^2}{63}\cdot 10^6 \Rightarrow v_2^2 = \left(64 - \frac{40\cdot 6{,}4^2}{63}\right)\cdot 10^6 \Rightarrow v_2 \simeq 6{,}16\cdot 10^3 \text{ m/s}.$$