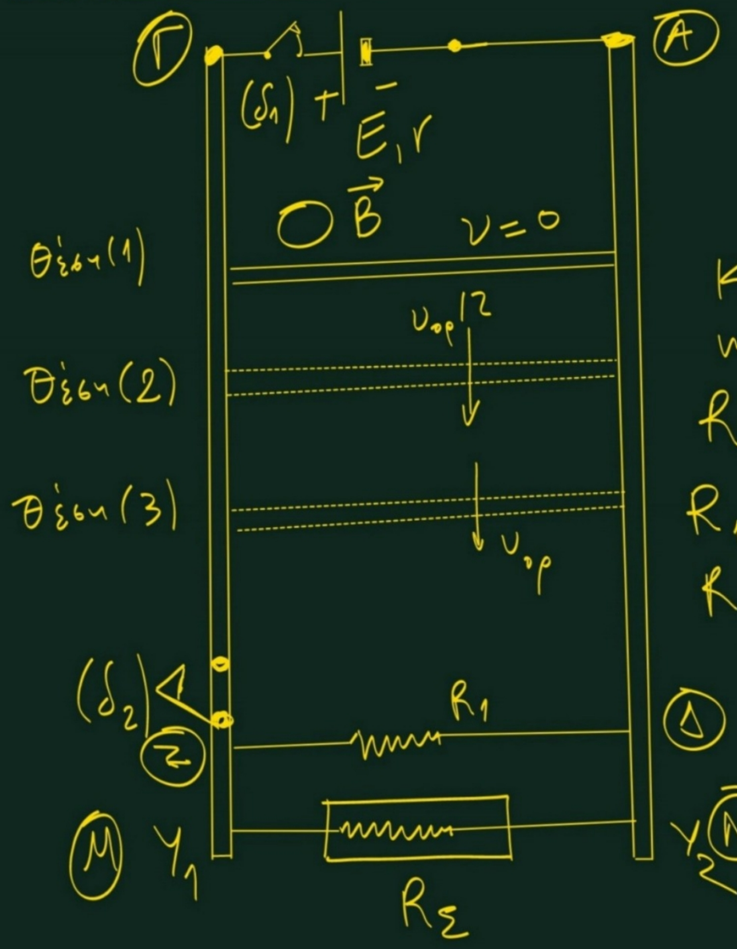
Ασκ. 5.202 (3ο Θέμα 2022)



$\ell = 1m$

$E = 9\,Volt$

$r = 1\Omega$

$KΛ: \ell = 1m$

$m = 0{,}3\,kg$

$R_{KΛ} = 2\Omega$

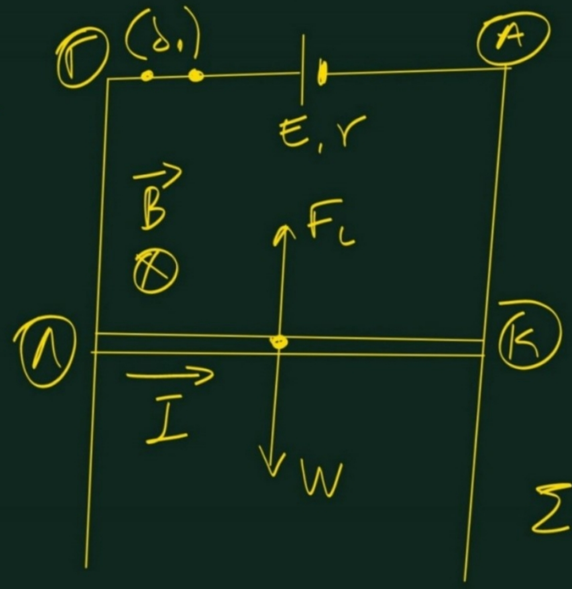
$R_1 = 3\Omega$

R_Σ

$V_\Sigma = 6\,Volt$

$P_\Sigma = 6\,Watt$

α) Αρχικά ο (δ1) είναι κλειστός και ο (δ2) ανοικτός.



$$I = \frac{E}{R_{KΛ} + r} \Rightarrow$$

$$I = \frac{9}{2+1} = 3A.$$

$$F_L = BI\ell = B \cdot 3 \cdot 1 \Rightarrow$$

$$F_L = 3B$$

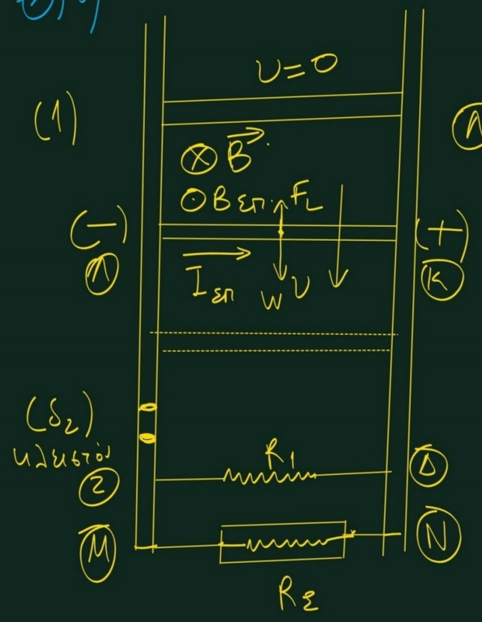
$$W = mg = 0{,}3 \cdot 10 = 3N$$

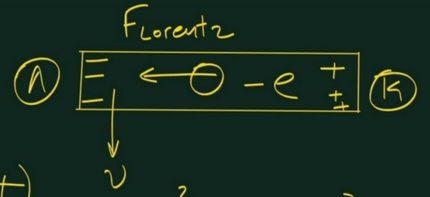
$$\Sigma F = 0 \Rightarrow F_L = W \Rightarrow$$

$$3B = 3 \Rightarrow B = 1\,Tesla$$

με φορά προς τα μέσα.

B) i)

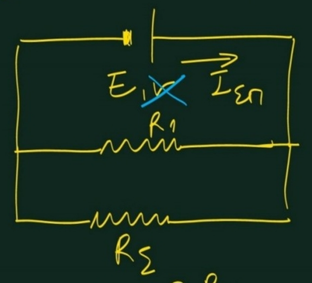




$$P_\Sigma = \frac{V_\Sigma^2}{R_\Sigma} \Rightarrow 6 = \frac{6^2}{R_\Sigma} \Rightarrow$$

$$R_\Sigma = 6\,\Omega$$

Ισοδύναμο κύκλωμα:



$$E = B\upsilon l$$

$$r \rightarrow R_{K\Lambda} = 2\,\Omega$$

$$R_{ολ} = R_{K\Lambda} + \frac{R_1 R_\Sigma}{R_1 + R_\Sigma} = 2 + \frac{3\cdot 6}{3+6} \Rightarrow$$

$$R_{ολ} = 2 + 2 = 4\,\Omega$$

Για τον ΚΛ ισχύει:

$$\Sigma F = W - F_L \Rightarrow ma = mg - BI_{ση}l = mg - B\frac{E}{R_{ολ}}\cdot l \Rightarrow$$

$$ma = mg - B\frac{Bl\upsilon}{R_{ολ}}\cdot l \Rightarrow ma = mg - \frac{B^2l^2}{R_{ολ}}\cdot\upsilon$$

(ii) Στη θέση (2) έχουμε:

$$\Sigma F = mg - B I_{\varepsilon\pi} \cdot \ell \Rightarrow$$

$$\frac{\Delta p}{\Delta t} = mg - B \frac{B \ell \upsilon}{R_{o\lambda}} \cdot \ell \Rightarrow$$

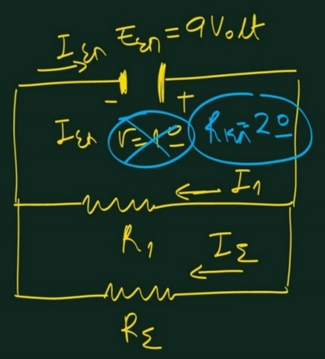
$$\frac{\Delta p}{\Delta t} = mg - \frac{B^2 \ell^2 \upsilon_{o\rho}}{2 R_{o\lambda}} \Rightarrow$$

$$\frac{\Delta p}{\Delta t} = 0{,}3 \cdot 10 - \frac{1 \cdot 1 \cdot 12}{2 \cdot 2} = 3 - 1{,}5 \Rightarrow$$

$$\frac{\Delta p}{\Delta t} = 1{,}5\ N \ \text{ή}\ kg \cdot m/s^2$$

(iii) Όταν η ταχύτητα γίνει οριακή η τάση στα άκρα του αγωγού σταθεροποιείται.

$$I_{\varepsilon\pi} = \frac{B \ell \cdot \upsilon_{o\rho}}{R_{o\lambda}} = \frac{1 \cdot 1 \cdot 12}{2} = 3A$$



$$I_1 \cdot R_1 = I_\Sigma R_\Sigma \Rightarrow$$

$$3 I_1 = 6 I_\Sigma \Rightarrow$$

$$I_1 = 2 I_\Sigma$$

$$I_1 + I_\Sigma = I_{\varepsilon\pi} \Rightarrow$$

$$3 I_\Sigma = 3 \Rightarrow I_\Sigma = 1A.$$

$$V_\Sigma = I_\Sigma \cdot R_\Sigma = 1 \cdot 6 = 6\ Volt$$

άρα λειτουργεί κανονικά.

$$\alpha = g - \frac{B^2 l^2}{m \cdot R_{ολ}} \cdot v$$

Άρα η κίνηση είναι ευθύγραμμη με σταδιακά επιταχυνόμενη με διαρκώς μειούμενη επιτάχυνση μέχρι η ταχύτητα να γίνει μέγιστη (οριακή και σταθερή).

Αν $\alpha = 0$ έχουμε: $g = \frac{B^2 l^2}{m R_{ολ}} \cdot v_{op} \Rightarrow$

$$v_{op} = \frac{mg \cdot R_{ολ}}{B^2 l^2} \Rightarrow v_{op} = \frac{0,3 \cdot 10 \cdot 6}{1^2 \cdot 1^2} \Rightarrow$$

$$v_{op} = 18 \, m/sec$$

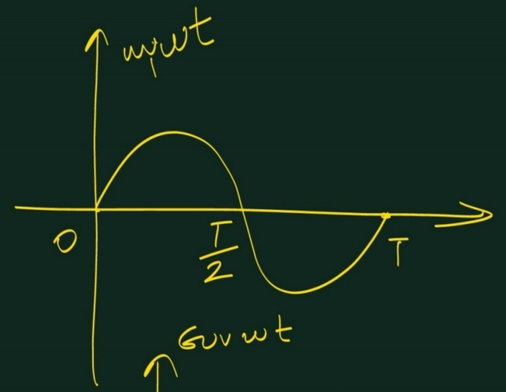
$i(t) = I \cdot \eta\mu\omega t$

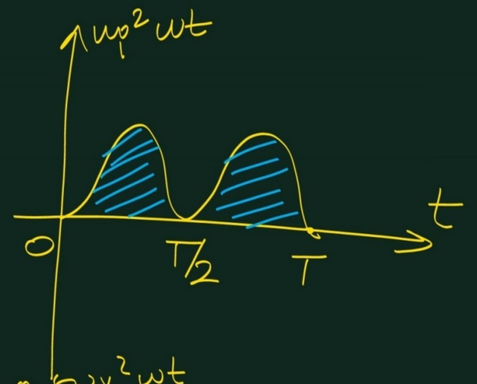
$v(t) = V \, \eta\mu\omega t$

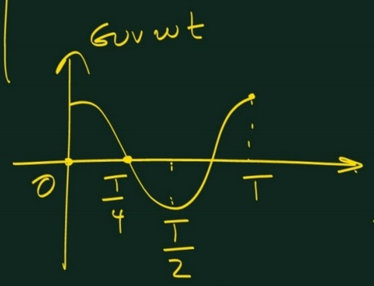
$V = I \cdot R$

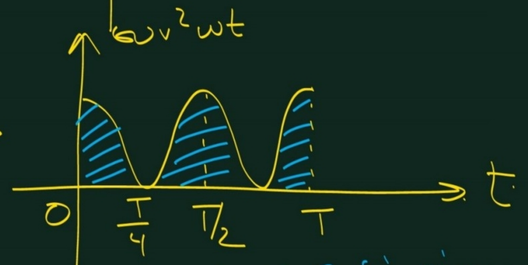
$p = i^2 R = I^2 \cdot \eta\mu^2\omega t \cdot R$

p = στιγμιαία ισχύς.









Τα γπλε εμβαδά είναι ίσα.

Άρα: $\dfrac{\Delta Q}{\Delta t} = I^2 R \cdot \eta\mu^2\omega t \Rightarrow$

$\Delta Q = I^2 \cdot R \cdot \eta\mu^2\omega t \cdot \Delta t \Rightarrow$

$\sum_{t=0}^{T} \Delta Q = I^2 \cdot R \cdot \sum_{t=0}^{T} \eta\mu^2\omega t \cdot \Delta t \Rightarrow Q = I^2 \cdot R \cdot \frac{1}{2} T$

Άρα: $\underbrace{\sum_{0}^{T} \eta\mu^2\omega t \cdot \Delta t = \sum_{0}^{T} \sigma\upsilon\nu^2\omega t \cdot \Delta t}_{\text{γπλε εμβαδά.}}$

$$\eta\mu^2\omega t + \sigma\upsilon\nu^2\omega t = 1 \Rightarrow$$

$$\sum_0^T \eta\mu^2\omega t\,\Delta t + \sum_0^T \sigma\upsilon\nu^2\omega t\,\Delta t = \sum_0^T \Delta t \Rightarrow$$

$$2\sum_0^T \eta\mu^2\omega t\cdot\Delta t = T \Rightarrow$$

$$\sum_0^T \eta\mu^2\omega t\cdot\Delta t = \frac{T}{2} = \sum_0^T \sigma\upsilon\nu^2\omega t\cdot\Delta t$$

Άρα η θερμότητα στο συνολλασσόμενο ανά περίοδο είναι:

$$Q = I^2\cdot R\cdot\frac{T}{2} = I_{\varepsilon\upsilon}^2 R T \Rightarrow I_{\varepsilon\upsilon}^2 = \frac{I^2}{2} \Rightarrow I_{\varepsilon\upsilon} = \frac{I}{\sqrt{2}}$$

$P = \upsilon\cdot i$ (στιγμιαία ισχύς)

$$P = I\cdot V\cdot\eta\mu^2\omega t = P(t)$$

$$W = \sum_0^T P(t)\,\Delta t =$$

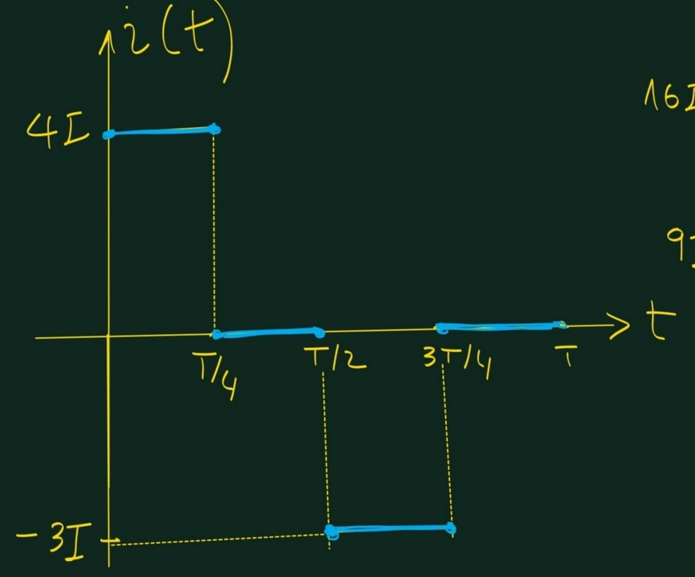
$$= \sum_0^T IV\,\eta\mu^2\omega t\,\Delta t = IV\cdot\frac{T}{2} \Rightarrow$$

$$\frac{W}{T} = \frac{IV}{2} \Rightarrow$$

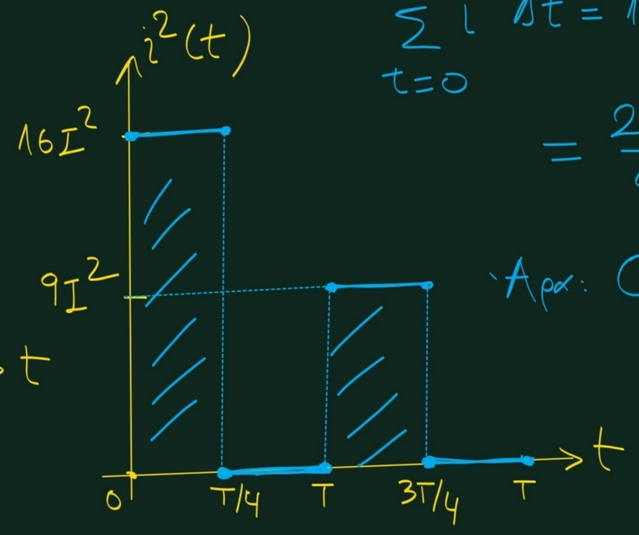
$$\bar{P} = I_{\varepsilon\upsilon}\cdot V_{\varepsilon\upsilon}$$

Ασκ. 6.68

Σε αντιστάτη υπάρχει μεταβαλλόμενο ρεύμα της μορφής



$$Q = \sum_{t=0}^{\Delta t} i^2 R \cdot \Delta t = I_{\varepsilon\nu}^2 R T$$



$$\sum_{t=0}^{T} i^2 \Delta t = 16 I^2 \cdot \frac{T}{4} + 9 I^2 \cdot \frac{T}{4} =$$

$$= \frac{25}{4} I^2 T$$

Άρα: $Q = \frac{25}{4} I^2 R T =$

$$= I_{\varepsilon\nu}^2 R T \Rightarrow$$

$$I_{\varepsilon\nu}^2 = \frac{25}{4} I^2 \Rightarrow$$

$$I_{\varepsilon\nu} = \frac{5}{2} I$$

Ασκ. 5.50 (σελ. 221)

$i = I\,\eta\mu 314t$

$\bar{P} = 100W$

α) Στιγμιαία ισχύς την $t_1 = 5\cdot 10^{-3}$ sec

$\omega = 314$ rad/sec

$2\pi f = 314 \Rightarrow$

$f = \frac{100\pi}{2\pi} = 50Hz$

$P = iv = i^2 R \Rightarrow$

$P = I^2 R \cdot \eta\mu^2 \omega t \Rightarrow$

$P = 2\bar{P}\cdot \eta\mu^2 \omega t \Rightarrow P = 2\cdot 100 \cdot \eta\mu^2 (314\cdot 5\cdot 10^{-3}) \Rightarrow$

$P = 200\cdot \eta\mu^2 (500\pi \cdot 10^{-3}) \Rightarrow$

$P = 200\,\eta\mu^2 \left(\frac{500\pi}{1000}\right) \Rightarrow$

$P = 200\,\eta\mu^2 \frac{\pi}{2} \Rightarrow P = 200$ Watt.

$\bar{P} = I_{\varepsilon\varphi} \cdot V_{\varepsilon\varphi} = \frac{I\cdot V}{2} \Rightarrow$

$\bar{P} = \frac{I^2 R}{2} \Rightarrow I^2 R = 2\bar{P}$

β) $\bar{P} = I_{\varepsilon\varphi}\cdot V_{\varepsilon\varphi} \Rightarrow 100 = 0{,}4\cdot V_{\varepsilon\varphi} \Rightarrow V_{\varepsilon\varphi} = 250$ Volt $\Rightarrow$

$\frac{V}{\sqrt{2}} = 250 \Rightarrow V = 250\sqrt{2}$ Volt.

$v(t) = V\cdot \eta\mu\omega t \Rightarrow v(t_2) = V\,\eta\mu\omega t_2$

Apα: $v(t_2) = 250\sqrt{2} \cdot \sin(314 \cdot 2{,}5 \cdot 10^{-3}) \Rightarrow$

$v(t_2) = 250\sqrt{2} \cdot \sin(100\pi \cdot 2{,}5 \cdot 10^{-3}) \Rightarrow$

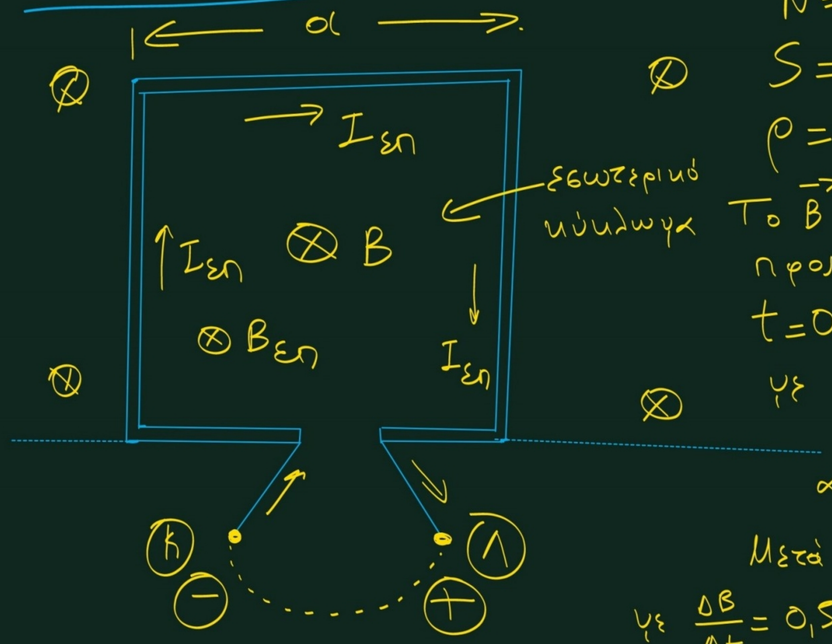
$v(t_2) = 250\sqrt{2} \sin\left(\frac{250\pi}{1000}\right) \Rightarrow$

$v(t_2) = 250\sqrt{2} \cdot \sin\frac{\pi}{4} \Rightarrow$

$v(t_2) = 250\sqrt{2} \cdot \frac{\sqrt{2}}{2} \Rightarrow$

$v(t_2) = 250 \text{ Volt}$

Ασκ. 5.195.



$\alpha = 20\,cm = 2\cdot10^{-1}\,m$

$N = 10$ σπείρες

$S = 2\,mm^2 = 2\cdot10^{-6}\,m^2$

$\rho = 10^{-6}\,\Omega\cdot m$

Το $\vec{B}$ έχει φορά πάντα προς τα γέσα.

$t=0$ ελαττώνεται με $\left|\frac{\Delta B}{\Delta t}\right| = 0{,}5\,T/sec$ από $B_0 = 2T$.

Μετά αυξάνεται με $\frac{\Delta B}{\Delta t} = 0{,}5\,T/sec$ μέχρι $B_2 = 4$ Tesla

α) Το $\vec{B}$ έχει φορά προς τα γέσα και γεμώνεται.

Άρα το $I_{επ}$ έχει τη δεξιόστροφη φορά που φαίνεται στο σχήμα.

Αλλά το πλαίσιο αποτελεί το εσωτερικό κύκλωμα, οπότε το υποθέμενο $I_{επ}$ πηγαίνει από το − στο +.

Άρα: − → K

και + → Λ

$$E_{επ} = N\cdot\left|\frac{\Delta\Phi}{\Delta t}\right| = 10\cdot0{,}5\cdot\alpha^2 = 5\cdot4\cdot10^{-2} = 0{,}2\,Volt$$

$$R_{\Pi} = \rho \cdot \frac{\ell}{S} = 10^{-6} \cdot \frac{N \cdot 4a}{2 \cdot 10^{-6}} = \frac{10 \cdot 4 \cdot 2 \cdot 10^{-1}}{2} = 4\,\Omega$$

i) $$Q = N \cdot \frac{|\Delta\Phi_1|}{R_{\Omega}} + N \cdot \frac{|\Delta\Phi_2|}{R_{\Omega}} = \frac{N}{R_{\Omega}} \left(|0 - B_0 a^2| + |B_2 a^2 - 0| \right) \Rightarrow$$

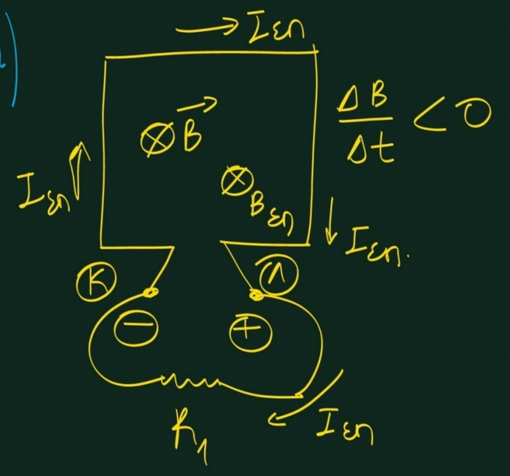
$$Q = \frac{N}{R_{\Omega}} \cdot a^2 (B_0 + B_2) = \frac{10}{R_{\Pi} + R_1} \cdot (2 \cdot 10^{-1})^2 \cdot (2+4) = \frac{10}{10} \cdot 4 \cdot 10^{-2} \cdot 6 \Rightarrow$$

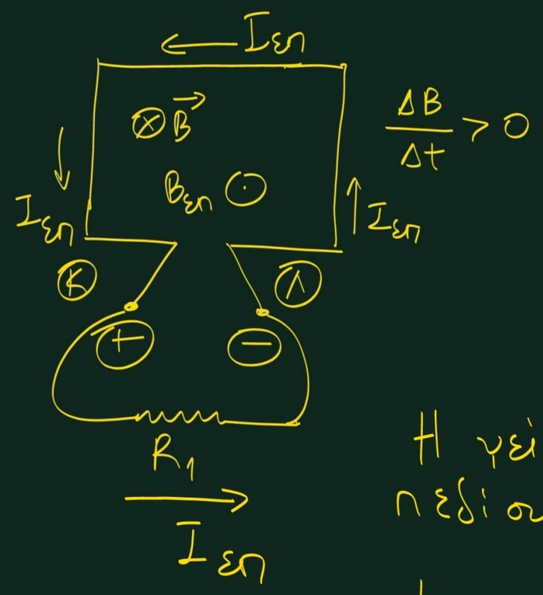
$$Q = 0{,}24\ Cb.$$

$$Q = -N \frac{\Delta\Phi_1}{R_{\Omega}} - N \frac{\Delta\Phi_2}{R_{\Omega}} = -\frac{N}{R_{\Omega}} (\Delta\Phi_1 + \Delta\Phi_2) = -\frac{N}{R_{\Omega}} \left((0 - B_0 a^2) + (B_2 a^2 - 0) \right) =$$

$$= -\frac{N}{R_{\Omega}} \cdot (B_2 - B_0) a^2 = -\frac{10}{10} \cdot (4-2) \cdot 4 \cdot 10^{-2} = -2 \cdot 10^{-2} = -8 \cdot 10^{-2}\ Cb$$

ii)





$$I_{\varepsilon\pi} = \frac{E_{\varepsilon\pi}}{R_1 + R_{\pi}} = \frac{0,2}{10} = 0,02\,A.$$

$$V_\Lambda - I_{\varepsilon\pi} R_1 = V_K \Rightarrow V_K - V_\Lambda = -I_{\varepsilon\pi} R_1 \Rightarrow$$

$$V_K - V_\Lambda = -0,12\,Volt$$

Επίσης: $V_{K\Lambda} = V_K - V_\Lambda$

$$V_K - I_{\varepsilon\pi} R_1 = V_\Lambda \Rightarrow$$

$$V_K - V_\Lambda = I_{\varepsilon\pi} R_1 \Rightarrow$$

$$V_K - V_\Lambda = +0,12\,Volt$$

Η μείωση του μαγνητικού πεδίου διαρκεί χρόνο t_1:

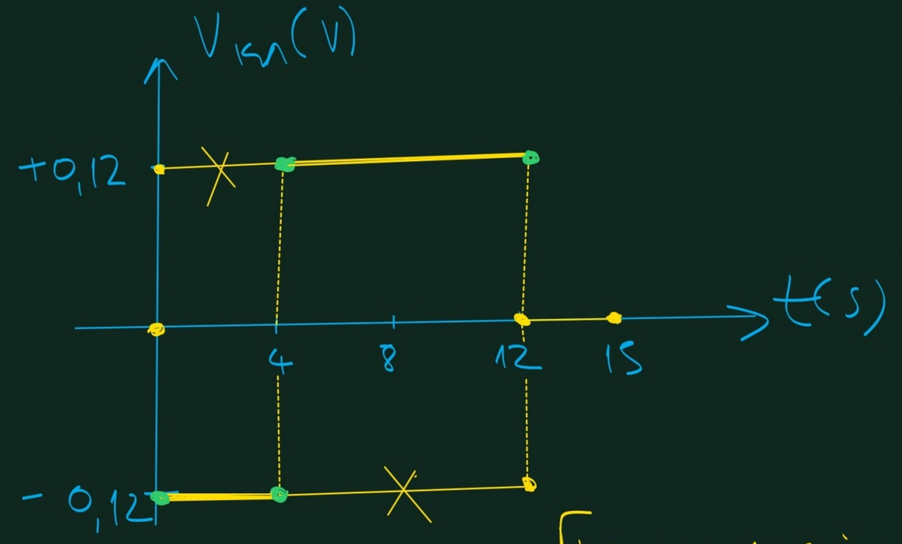
$$t_1 = \frac{2}{0,5} = 4\,sec$$

και η αύξηση διαρκεί χρόνο t_2:

$$t_2 = \frac{4}{0,5} = 8\,sec$$

Άρα:

$$V_{KA} = \begin{cases} -0{,}12\ \text{Volt} & \text{αν } 0 \le t < 4\,\text{sec} \\ +0{,}12\ \text{Volt} & \text{αν } 4 \le t < 12\,\text{sec} \\ 0\ \text{Volt} & \text{αν } 12 \le t \le 15\,\text{sec} \end{cases}$$



iii) Για το πλαίσιο:

$$Q_n = I_{\varepsilon n}^2 R_n \cdot t_1 + I_{\varepsilon n}^2 R_n \cdot t_2 =$$

$$= I_{\varepsilon n}^2 R_n (t_1 + t_2) =$$

$$= 0{,}02^2 \cdot 4 \cdot 12 = 4 \cdot 10^{-4} \cdot 48 \Rightarrow$$

$$Q_n = 192 \cdot 10^{-4} \Rightarrow Q_n = 1{,}92 \cdot 10^{-2}\ \text{Joule}$$

Για τον αντιστάτη R_1: $Q_1 = I_{\varepsilon n}^2 R_1 (t_1 + t_2) = 4 \cdot 10^{-4} \cdot 6 \cdot 12 =$

$$= 2{,}88 \cdot 10^{-2}\ \text{Joule}$$

Αυτεπαγωγή.

$$E_{αυτ} = -L\frac{\Delta i}{\Delta t}$$

Η πτώση τάσης στο πηνίο:

$$V_L = L\cdot\frac{\Delta i}{\Delta t}$$

Άρα: $E_{αυτ} = -V_L$

Μαγνητικό πεδίο πηνίου:

$$B = \mu\mu_0 n i$$

$$U_L = \frac{1}{2}Li^2 \longrightarrow K = \frac{1}{2}mv^2$$

$$E_{αυτ} = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{\Delta(BA)}{\Delta t} =$$

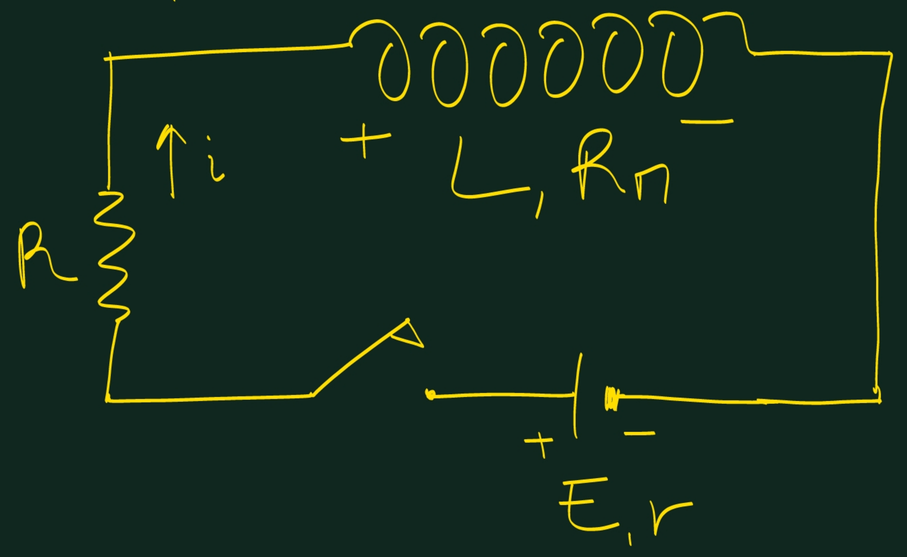
$$= -NA\frac{\Delta B}{\Delta t} = -NA\frac{\Delta(\mu\mu_0 n i)}{\Delta t} =$$

$$= -NA\mu\mu_0 n\frac{\Delta i}{\Delta t} = -L\frac{\Delta i}{\Delta t}$$

Άρα: $L = NA\mu\mu_0 n$

όπου $n = \frac{N}{\ell}$ και A = εμβαδό διατομής πηνίου.

Καμπύλη «φόρτισης» πηνίου.



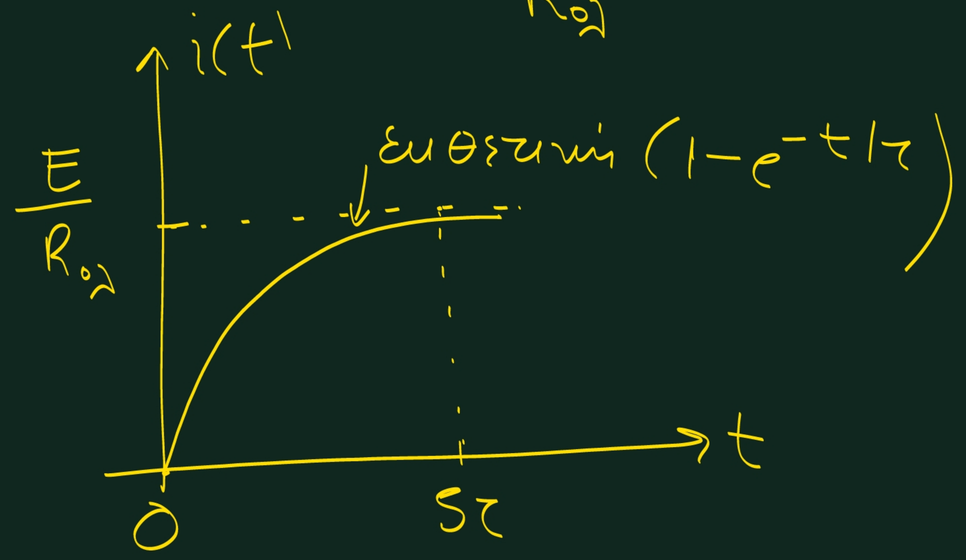
$$E = iR + iR_\pi + ir + V_L \Rightarrow$$

$$E = i(R + R_\pi + r) + L\frac{\Delta i}{\Delta t} \Rightarrow$$

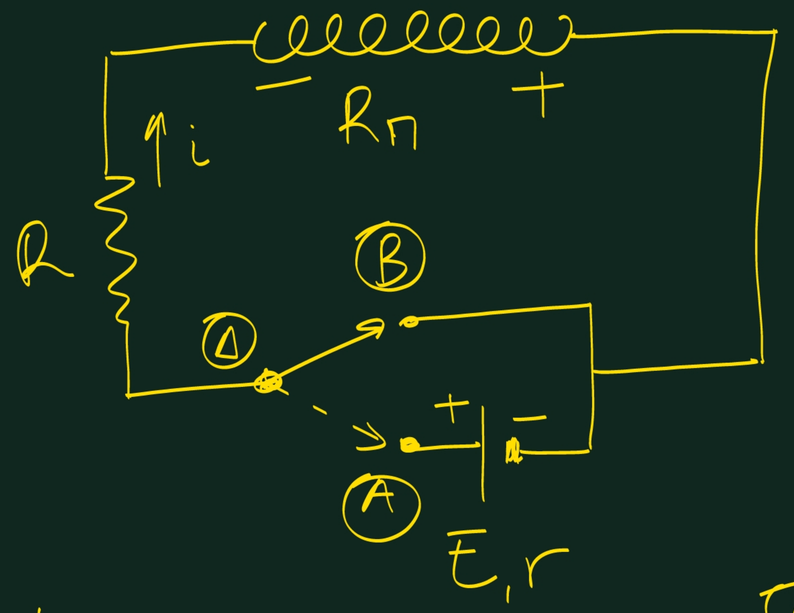
$$E = iR_{ολ} + L\cdot\frac{\Delta i}{\Delta t} \Rightarrow (\text{ΚΝΥΛ})$$

$$\frac{E}{R_{ολ}} = i + \frac{L}{R_{ολ}}\cdot\frac{\Delta i}{\Delta t} \qquad \tau = \frac{L}{R_{ολ}}.$$

Αποτέλεσμα: $i = \frac{E}{R_{ολ}}\left(1 - e^{-t/\tau}\right)$



Καμπύλη «εκφόρτισης» πηνίου



Όταν ο γεφυρωτής ήταν στο (A) το ρεύμα στο πηνίο είχε πάρει την τελική του τιμή: $I = \dfrac{E}{R+R_\pi+r}$

Αν γυρίσουμε τον γεφυρωτή στο (B), το πηνίο θα αρχίσει να «εκφορτίζεται» μέσω των αντιστάσεων R και R_π.

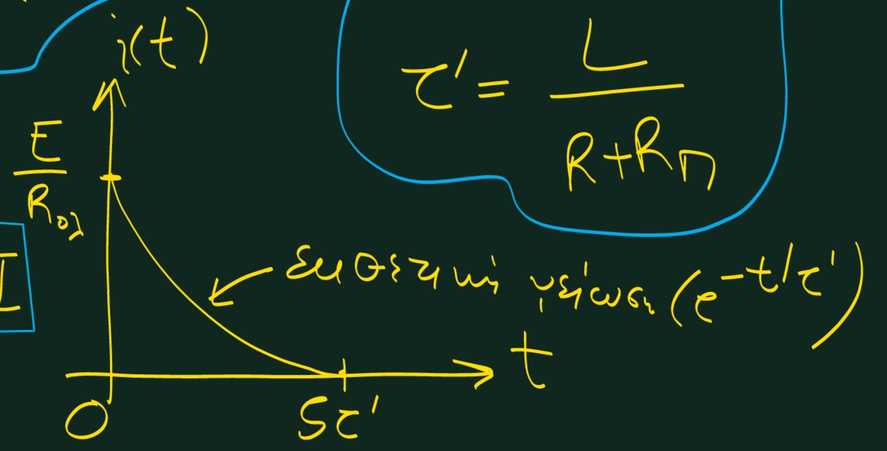
KVL: $E_{\alpha\nu\tau} = iR + iR_\pi \Rightarrow$

$$-L\cdot\frac{\Delta i}{\Delta t} = i(R+R_\pi) \Rightarrow i(R+R_\pi) + L\frac{\Delta i}{\Delta t} = 0 \Rightarrow$$

$$\frac{L}{R+R_\pi}\frac{\Delta i}{\Delta t} + i = 0 \Rightarrow i(t) = i(0)e^{-t/\tau'}$$

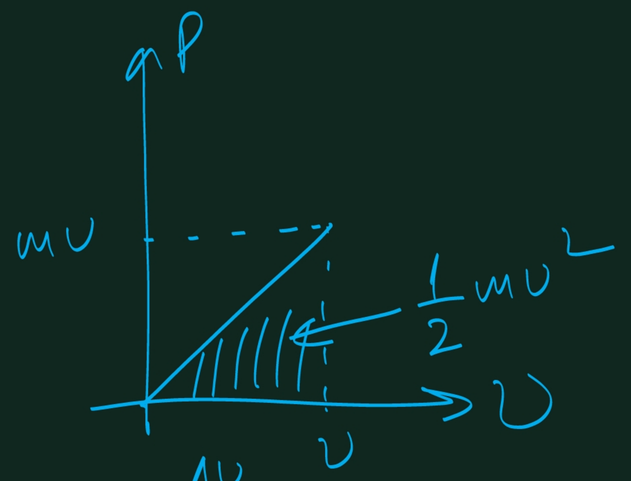
$$\tau' = \frac{L}{R+R_\pi}$$

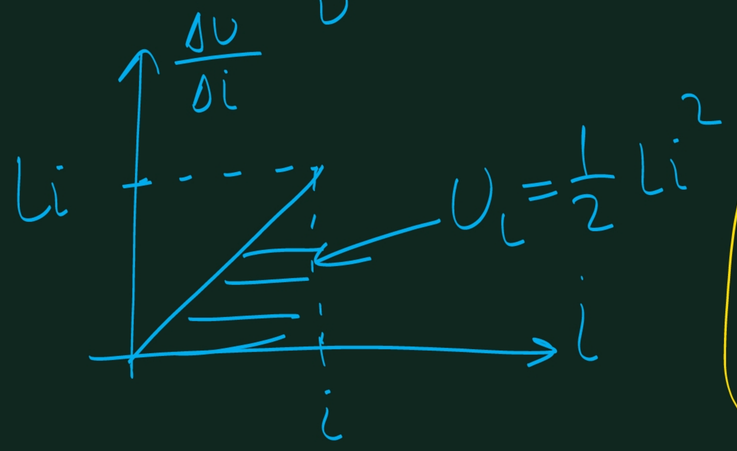
$$\boxed{i(0) = I}$$



$$P = m \cdot v = \frac{\Delta K}{\Delta v}$$

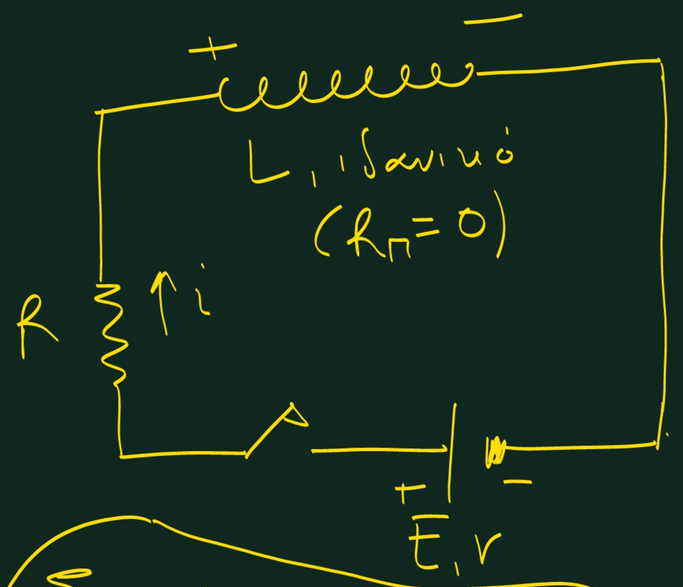
$$\frac{\Delta U}{\Delta i} = L \cdot i$$





Ασκ. 5.68 (σελ. 226)

«Φόρτιση πηνίου»



α) Στη γόνιμη απόμειση το πηνίο συμπεριφέρεται σαν βραχυκύκλωμα.

Άρα: $E = IR + I \cdot r \Rightarrow$

$$I = \frac{E}{R+r} = \frac{40}{8+2} = 4\,A.$$

ή αλλιώς:

$$E = iR + ir + L\frac{\Delta i}{\Delta t} \Rightarrow$$

$$\frac{E}{R+r} = i + \frac{L}{R+r}\frac{\Delta i}{\Delta t}$$

Στη γόνιμη απόμειση

$\frac{\Delta i}{\Delta t} = 0$ και $i \to I$

Άρα: $\frac{E}{R+r} = I$