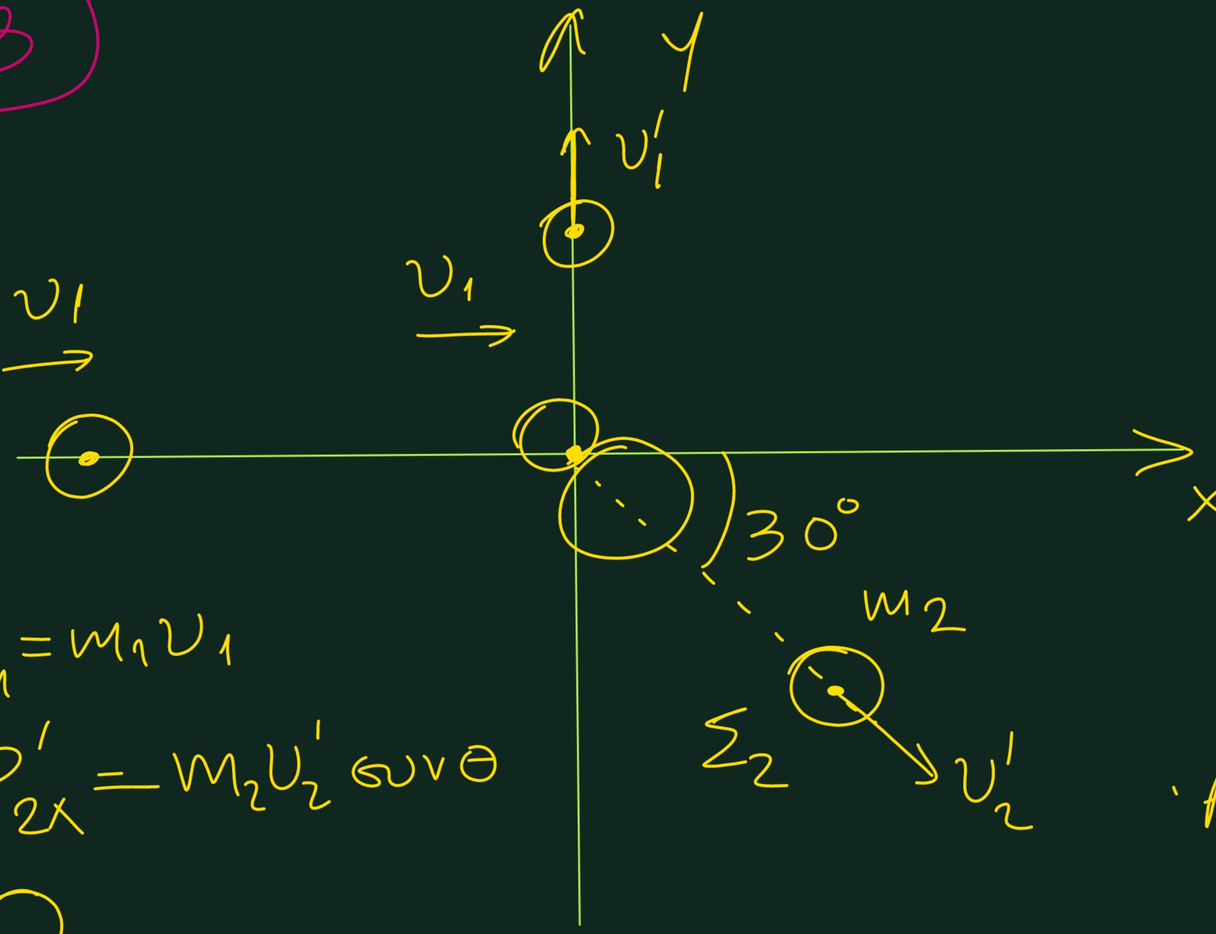
Θέμα Β3

$m_1 = m$

$m_2 = 2m$

$m_3 = m$

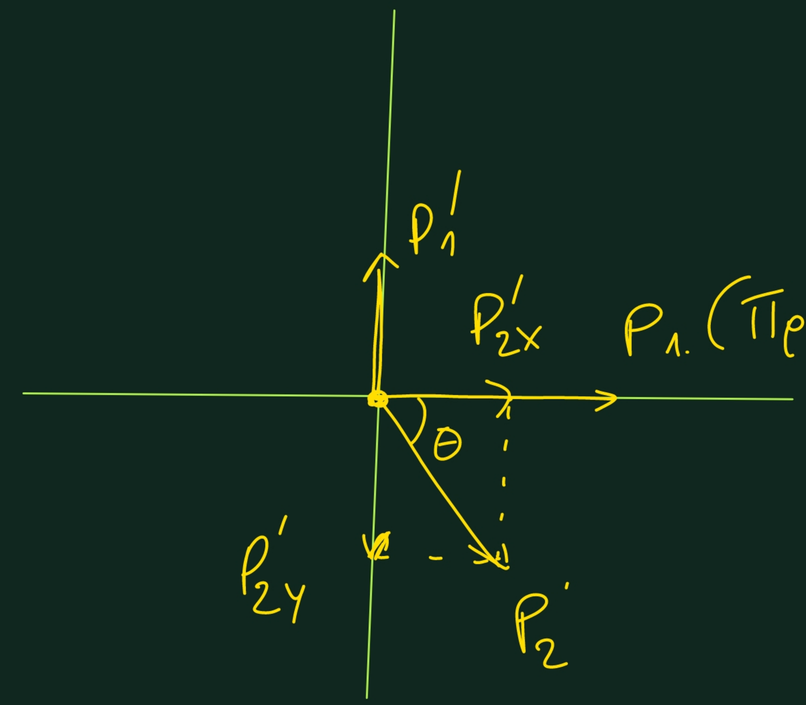


$p_{x,\pi\rho\iota\nu} = p_1 = m_1 v_1$

$p_{x,\mu\varepsilon\tau\acute{\alpha}} = p_{2x}' = m_2 v_2' \sigma\upsilon\nu\theta$

$p_{y,\pi\rho\iota\nu} = 0$

$p_{y,\mu\varepsilon\tau\acute{\alpha}} = p_1' - p_{2y}' = m_1 v_1' - m_2 v_2' \eta\mu\theta$



Άρα: $m_1 v_1 = m_2 v_2' \sigma\upsilon\nu\theta \Rightarrow$

$m v_1 = 2m v_2' \cdot \frac{\sqrt{3}}{2} \Rightarrow$

$v_2' = \frac{v_1}{\sqrt{3}}$

$p_{y,\text{πριν}} = p_{y,\text{μετά}} \Rightarrow$

$0 = m_1 v_1' - m_2 v_2' \eta\mu\theta \Rightarrow$

$m_1 v_1' = m_2 v_2' \eta\mu\theta \Rightarrow$

$m v_1' = 2m \frac{v_1}{\sqrt{3}} \cdot \frac{1}{2} \Rightarrow$

$v_1' = \frac{v_1}{\sqrt{3}}$

Πλαστική κρούση:

$m_1 v_1' = (m_1 + m_3) V \Rightarrow$

$m_1 \cdot \frac{v_1}{\sqrt{3}} = 2m V \Rightarrow V = \frac{v_1}{2\sqrt{3}}$

$$\frac{K_{\text{συσ}}}{K_1} = \frac{\frac{1}{2} \cdot 2m V^2}{\frac{1}{2} m_1 v_1^2} = \frac{\frac{m v_1^2}{4 \cdot 3}}{\frac{1}{2} m v_1^2} = \frac{2}{4 \cdot 3} = \frac{1}{6}$$

Σωστό το (iii).

Κεντρική ελαστική κρούση.

Από $\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2=\frac{1}{2}m_1v_1'^2+\frac{1}{2}m_2v_2'^2$

και $m_1v_1+m_2v_2=m_1v_1'+m_2v_2'$

καταλήγουμε στην:

$$v_1'-v_2'=-v_1+v_2$$

Τα v_1' και v_2' τα βρίσκουμε λύνοντας το σύστημα:

$$\begin{cases} m_1v_1'+m_2v_2'=m_1v_1+m_2v_2 \\ v_1'-v_2'=-v_1+v_2 \end{cases} \begin{matrix} \cdot 1 \\ \cdot m_2 \end{matrix} \Rightarrow$$

$$m_1v_1'+m_2v_2'=m_1v_1+m_2v_2$$
$$m_2v_1'-m_2v_2'=-m_2v_2+m_2v_2 \quad (+)$$

$$(m_1+m_2)v_1'=(m_1-m_2)v_1+2m_2v_2 \Rightarrow$$

$$v_1'=\frac{m_1-m_2}{m_1+m_2}v_1+\frac{2m_2}{m_1+m_2}v_2$$

Ομοίως:

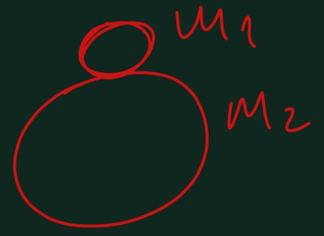
$$\begin{cases} m_1v_1'+m_2v_2'=m_1v_1+m_2v_2 \\ v_1'-v_2'=-v_1+v_2 \end{cases} \begin{matrix} \cdot 1 \\ \cdot (-m_1) \end{matrix} \Rightarrow$$

$$m_1v_1'+m_2v_2'=m_1v_1+m_2v_2$$
$$-m_1v_1'+m_1v_2'=m_1v_1-m_1v_2 \quad (+)$$

$$(m_1+m_2)v_2'=2m_1v_1+(m_2-m_1)v_2 \Rightarrow$$

$$v_2'=\frac{2m_1v_1}{m_1+m_2}+\frac{(m_2-m_1)}{m_1+m_2}v_2$$

Ελαστική κρούση σώματος m_1 με άλλο σώμα $m_2 \gg m_1$



$m_2 \gg m_1$

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

$\Rightarrow$

$$v_1' \simeq \frac{-m_2}{m_2} v_1 + \frac{2m_2}{m_2} v_2$$

$$v_2' \simeq \frac{2m_1}{m_2} v_1 + \frac{m_2}{m_2} v_2$$

$\Rightarrow$

$$v_1' \simeq -v_1 + 2v_2$$

$$v_2' \simeq v_2$$

$\Rightarrow$

$v_1' = -v_1$

$v_2' = 0$

ακίνητος τοίχος