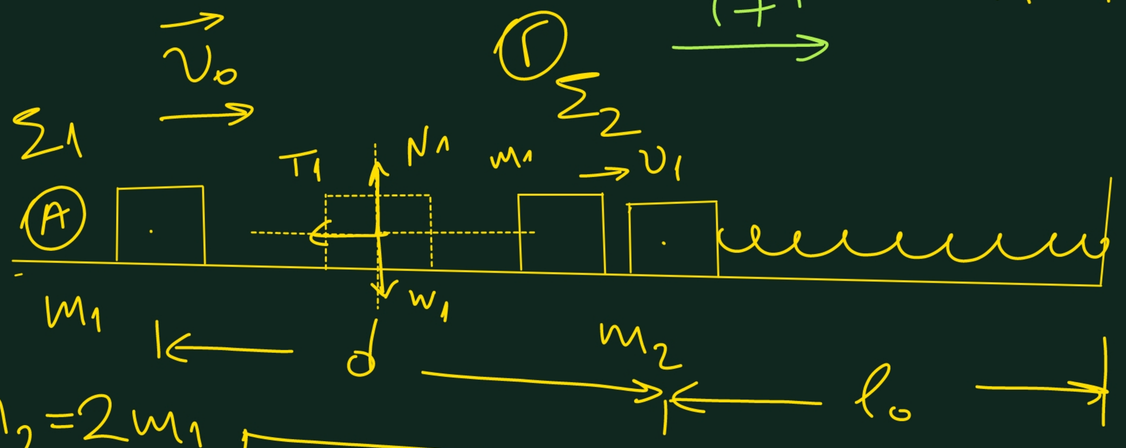
Ασκ. 2.84 (Παν 2013 Γ):



$m_2 = 2m_1$

$d = 1\,m$.

$\mu = 0{,}5$

$w_1 = N_1$

$T_1 = \mu N_1 = \mu w_1 = m_1 \mu g$.

α) Όταν το Σ_1 φτάνει στο m_2 έχει ταχύτητα v_1.

ΘΜΚΕ: (A) → (Γ):

$\Delta K_1 = \Sigma W \Rightarrow \frac{1}{2} m_1 v_1^2 - \frac{1}{2} m_1 v_0^2 = -T_1 \cdot d \Rightarrow$

$\frac{1}{2} m_1 v_1^2 - \frac{1}{2} m_1 v_0^2 = -\mu m_1 g d \Rightarrow v_1^2 - v_0^2 = -2\mu g d$

Μετά την πρόσκ.:

$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 \Rightarrow$

$-\sqrt{10} = \frac{m_1 - 2m_1}{m_1 + 2m_1} v_1 \Rightarrow v_1 = 3\sqrt{10}\ m/s$.

Άρα: $(3\sqrt{10})^2 - v_0^2 = -2 \cdot 0{,}5 \cdot 10 \cdot 1 \Rightarrow$

$90 - v_0^2 = -10 \Rightarrow v_0^2 = 100 \Rightarrow$

$v_0 = 10\ m/s$.

β) $K_1 = \frac{1}{2} m_1 v_1^2$

$K_2' = \frac{1}{2} m_2 v_2'^2$

Το ζητούμενο ποσοστό είναι:

$$\Pi\% = \frac{K_2'}{K_1}\cdot 100 = \frac{\frac{1}{2}m_2 v_2'^2}{\frac{1}{2}m_1 v_1^2}\cdot 100 = \frac{m_2 v_2'^2}{m_1 v_1^2}\cdot 100$$

$$v_2' = \frac{2m_1 v_1}{m_1+m_2} = \frac{2m_1 v_1}{3m_1} = \frac{2v_1}{3}$$

Άρα: $$\Pi\% = \frac{2m_1\cdot\left(\frac{2v_1}{3}\right)^2}{m_1 v_1^2}\cdot 100 = \frac{2m_1\cdot\frac{4v_1^2}{9}}{m_1 v_1^2}\cdot 100 = \frac{800}{9}\%$$

Στη συνέχεια:

$0 = |v_1'| - \alpha_1 t_2 \Rightarrow$

$\alpha_1 t_2 = |v_1'| \Rightarrow$

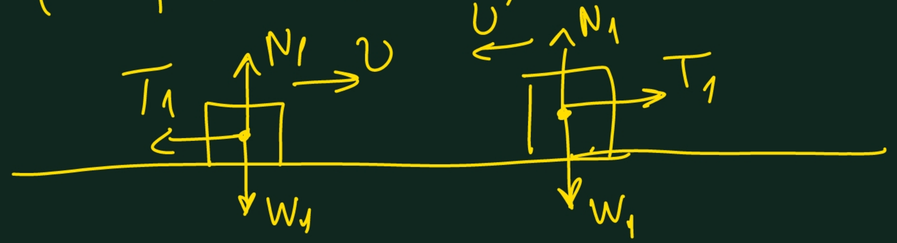
$5t_2 = \sqrt{10} \Rightarrow 5t_2 = 3{,}2 \Rightarrow$

$t_2 = 0{,}64\ sec$

Άρα: $t_{o\lambda} = t_1 + t_2 = 0{,}08 + 0{,}64$

$\Rightarrow \boxed{t_{o\lambda} = 0{,}72\ sec}$

γ) $|\Sigma F_1| = T_1 \Rightarrow m_1 a_1 = \gamma m_1 g \Rightarrow a_1 = \gamma g$ (επιβράδυνση)



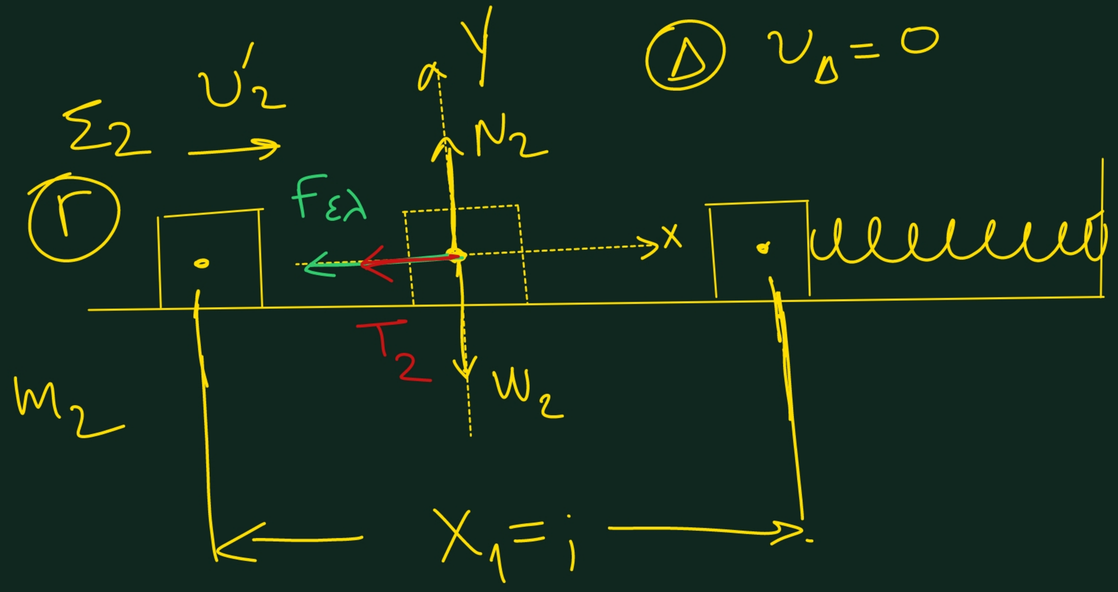
Αρχικά: $v_1 = v_0 - a_1 t_1 \Rightarrow 3\sqrt{10} = 10 - 0{,}5\cdot 10\cdot t_1 \Rightarrow$

$5t_1 = 10 - 3\sqrt{10} \Rightarrow 5t_1 = 10 - 3\cdot 3{,}2 \Rightarrow$

$5t_1 = 10 - 9{,}6 \Rightarrow 5t_1 = 0{,}4 \Rightarrow t_1 = 0{,}08\ sec.$

β) Μετά την κρούση το m_2 έχει ταχύτητα:

$$v_2' = \frac{2v_1}{3} \Rightarrow v_2' = \frac{2\sqrt{10}}{3}\ m/s.$$



$$W_2 = N_2$$

$$T_2 = \mu N_2 = \mu m_2 g.$$

ΘΜΚΕ: Γ → Δ:

$$0 - \frac{1}{2}m_2 v_2'^2 = -T_2 \cdot x_1 + W_{F_{ελ}(Γ\to Δ)} \Rightarrow$$

$$-\frac{1}{2}m_2 v_2'^2 = -\mu m_2 g x_1 + U_{ελ(Γ)} - U_{ελ(Δ)} \Rightarrow$$

$$-\frac{1}{2}m_2 v_2'^2 = -\mu m_2 g x_1 + 0 - \frac{1}{2}k x_1^2 \Rightarrow$$

$$-\frac{1}{2}\cdot\left(2\sqrt{10}\right)^2 = -0{,}5\cdot 1\cdot 10\cdot x_1 - \frac{1}{2}\cdot 105 x_1^2 \Rightarrow$$

$$-\frac{1}{2}\cdot 4\cdot 10 = -5x_1 - \frac{105}{2}x_1^2 \Rightarrow$$

$$105x_1^2 + 10x_1 - 40 = 0 \Rightarrow 21x_1^2 + 2x_1 - 8 = 0$$

$$\Delta = 4 + 4\cdot 21\cdot 8 = 4(1+168) = 4\cdot 169$$

$$21x_1^2 + 2x_1 - 8 = 0$$

$$\Delta = 4 \cdot 169.$$

$$x_1 = \frac{-2 \pm \sqrt{4 \cdot 169}}{2 \cdot 21} \Rightarrow$$

$$x_1 = \frac{-2 \pm 2 \cdot 13}{2 \cdot 21} = \frac{-2 \pm 26}{42}$$

Άρα: $x_1 = \frac{24}{42} = \frac{12}{21} = \frac{3 \cdot 4}{3 \cdot 7} \Rightarrow$

$$x_1 = \frac{4}{7} \text{ m.}$$