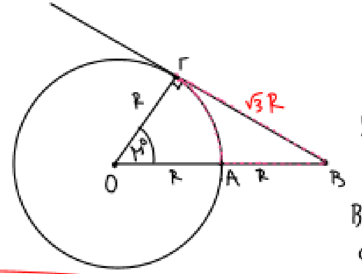
1) απδ. σελ 107



$\kappa(O,R)$, $OA = OB$

ΑΒΓ – καμπυλόγραμμο τρίγ.

ℓ; E;

ΛΥΣΗ

$\ell = \ell_{\overset{\frown}{A\Gamma}} + AB + B\Gamma$

$O\hat{\Gamma}B$ – ορθογώνιο

Βάσει του Π.Θ.

$OB^2 = O\Gamma^2 + B\Gamma^2 \Leftrightarrow B\Gamma^2 = OB^2 - O\Gamma^2 \Leftrightarrow$

$\Leftrightarrow B\Gamma^2 = (2R)^2 - R^2 \Leftrightarrow B\Gamma^2 = 3R^2 \Leftrightarrow B\Gamma = \sqrt{3}R$

$\ell_{\overset{\frown}{A\Gamma}} = \dfrac{\pi \cdot R \cdot \mu^\circ}{180^\circ}$

$O\hat{\Gamma}B$ – ορθογώνιο, $OB = 2O\Gamma$ } $\hat{B} = 30^\circ$, άρα $\mu^\circ = 60^\circ$ } $\ell_{\overset{\frown}{A\Gamma}} = \dfrac{\pi R}{3}$

$$\ell = \frac{\pi R}{3} + R + \sqrt{3}R$$

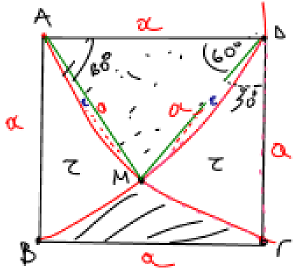
$E = (O\hat{\Gamma}B) - (O\overset{\frown}{A\Gamma})$

$(O\hat{\Gamma}B) = \dfrac{O\Gamma \cdot B\Gamma}{2} = \dfrac{R \cdot \sqrt{3}R}{2} = \dfrac{\sqrt{3}}{2}R^2$

$(O\overset{\frown}{A\Gamma}) = \dfrac{\pi \cdot R^2 \cdot 60^\circ}{360^\circ} = \dfrac{\pi R^2}{6}$

$$E = \frac{\sqrt{3}}{2}R^2 - \frac{\pi R^2}{6}$$

2



$E = (AB\Gamma\Delta) - 2\tau$ (1)

$(AB\Gamma\Delta) = \alpha^2$ (2)

$\tau = (\Delta \overset{\frown}{M\Gamma}) - \varepsilon$ (3)

$(\Delta \overset{\frown}{M\Gamma}) = \dfrac{\pi \cdot \alpha^2 \cdot 30^\circ}{360^\circ} = \dfrac{\pi \alpha^2}{12}$ (4)

$\varepsilon = (A\overset{\frown}{M\Delta}) - (A\hat{M}\Delta)$

$A\hat{M}\Delta$ – ισόπλευρο: $(A\hat{M}\Delta) = \dfrac{\alpha^2\sqrt{3}}{4}$

$(A\overset{\frown}{M\Delta}) = \dfrac{\pi \cdot \alpha^2 \cdot 60^\circ}{360^\circ} = \dfrac{\pi \alpha^2}{6}$

$\varepsilon = \dfrac{\pi\alpha^2}{6} - \dfrac{\alpha^2\sqrt{3}}{4}$ (5)

(3),(4),(5): $\tau = \dfrac{\pi\alpha^2}{12} - \left(\dfrac{\pi\alpha^2}{6} - \dfrac{\alpha^2\sqrt{3}}{4}\right)$ (6)

(1),(2),(6): $E = \alpha^2 - 2\left[\dfrac{\pi\alpha^2}{12} - \left(\dfrac{\pi\alpha^2}{6} - \dfrac{\alpha^2\sqrt{3}}{4}\right)\right]$